LOBACHEVSKY STATE UNIVERSITY
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National Research University

Generation of entangled microwave photons in superconducting circuits

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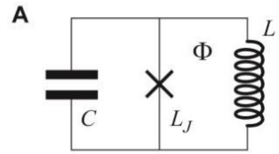
Outline

- Superconducting circuits for quantum information processing
- What are Josephson junctions? How do they work?
- A circuit analog for cavity QED
- Entanglement
- How to implement the microwave down-conversion effect in a waveguide with an embedded Josephson junction?
- Classical electrodynamics of a waveguide
- Quantum theory of microwave down conversion
- Summary

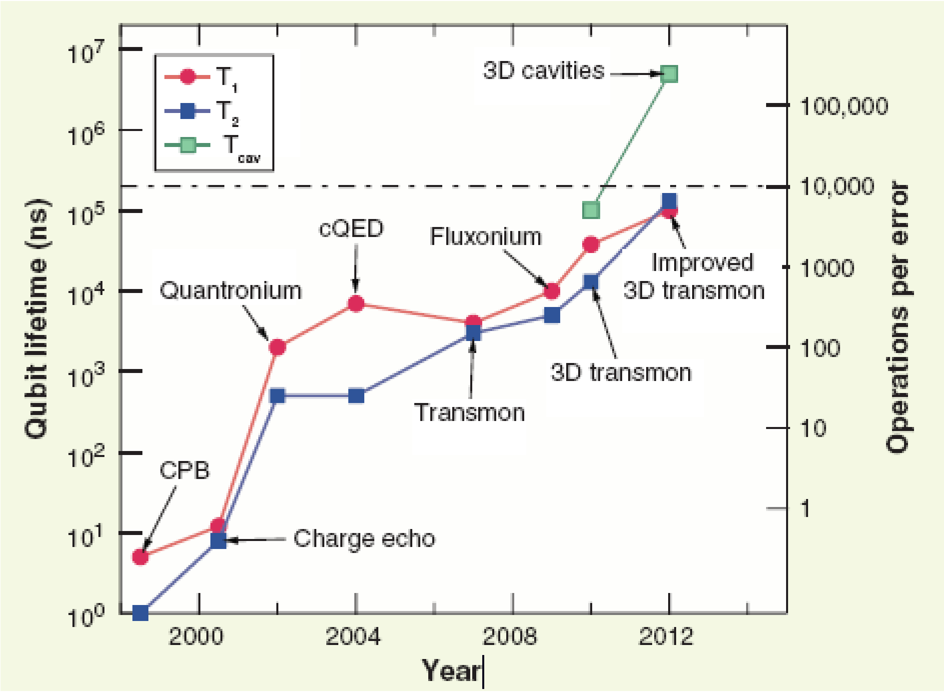
The Future of Quantum Information Processing

State of the art

Superconducting Circuits for Quantum Information: An Outlook, M. H. Devoret and R. J. Schoelkopf, *Science 339,1163(2013)*

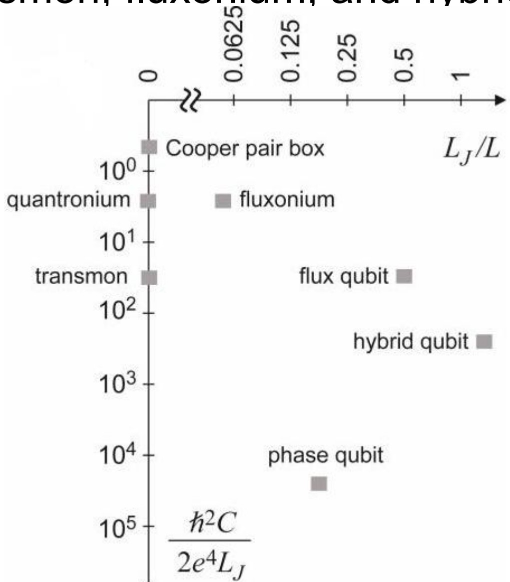


Josephson junction is in the heart of quantum information processing schemes



Examples of the "Moore's law" type of exponential scaling in performance
of superconducting qubits during recent years

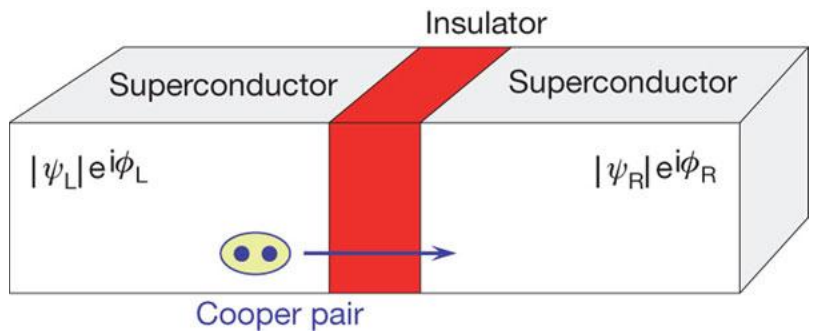
Mendeleev-like but continuous "table" of artificial atom types: Cooper pair box, flux qubit, phase qubit, quantronium, eransmon, fluxonium, and hybrid qubit



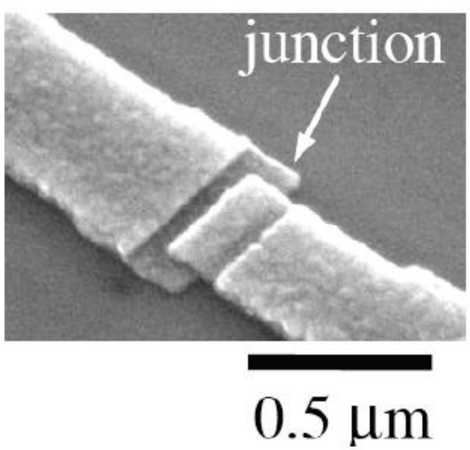
The horizontal and vertical coordinates correspond to fabrication parameters that determine the
inverse of the number of corrugations in the potential and the number of levels per well, respectively.

What are Josephson junctions?

A Josephson junction is composed of two bulk superconductors separated by a thin insulating layer through which Cooper pairs can tunnel.



B. D. Josephson, “Possible new effects in superconductive tunneling,” *Phys. Lett.*, Vol. 1, pp. 251–253, July 1962. P.

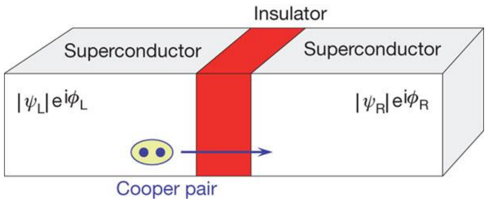


The devices are named after Brian **Josephson**, who predicted in 1962 that pairs of superconducting electrons could "tunnel" right through the nonsuperconducting barrier from one superconductor to another. He also predicted the exact form of the current and voltage relations for the junction. Experimental work proved that he was right, and Josephson was awarded the 1973 Nobel Prize in Physics for his work.

C.H. van der Wal. Technische Universiteit Delft, 2001.

How do they work?

R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics, Vol. III*



$$\begin{cases} i\hbar \dfrac{\partial \psi_L}{\partial t} = U_L \psi_L + K\psi_R, \\ i\hbar \dfrac{\partial \psi_R}{\partial t} = U_R \psi_R + K\psi_L. \end{cases} \qquad \begin{cases} \psi_L = |\psi_L| e^{i\varphi_L}, \\ \psi_R = |\psi_R| e^{i\varphi_R}. \end{cases}$$

The supercurrent through the junction is $I = I_c \sin\varphi,$ where the critical current $I_c = \dfrac{2K}{\hbar}\sqrt{|\psi_L| . |\psi_R|}$

$\varphi = \varphi_L - \varphi_R$ is the phase difference of the two superconductors across the junction

The time variation of this phase difference is related to the potential difference *V* between the two superconductors:

$$\frac{d\varphi}{dt} = \frac{2eV}{\hbar}$$

In summary:

$$\begin{cases} I = I_c \sin\varphi \\ \dfrac{d\varphi}{dt} = \dfrac{2eV}{\hbar} \end{cases}$$

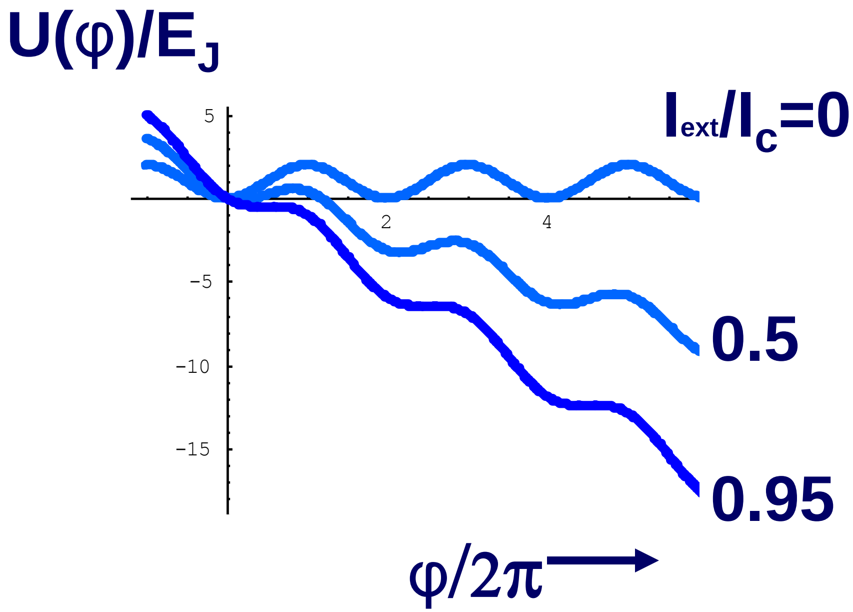
Energy of the junction:

$$\begin{cases} I = I_c \sin\varphi \\ \dfrac{d\varphi}{dt} = \dfrac{2eV}{\hbar} \end{cases}$$

$$H = \frac{CV^2}{2} + E_J(1-\cos\varphi) - \frac{\hbar}{2e} I_{ext}\varphi = \frac{J}{2}\left(\frac{d\varphi}{dt}\right)^2 + E_J(1-\cos\varphi) - \frac{\hbar}{2e} I_{ext}\varphi$$

$$J = C(\hbar/2e)^2$$

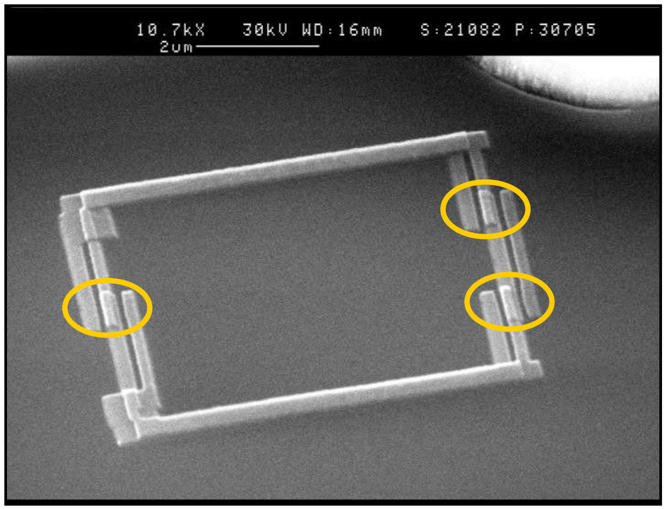
$$E_J = \left(\hbar/2e\right) I_c$$

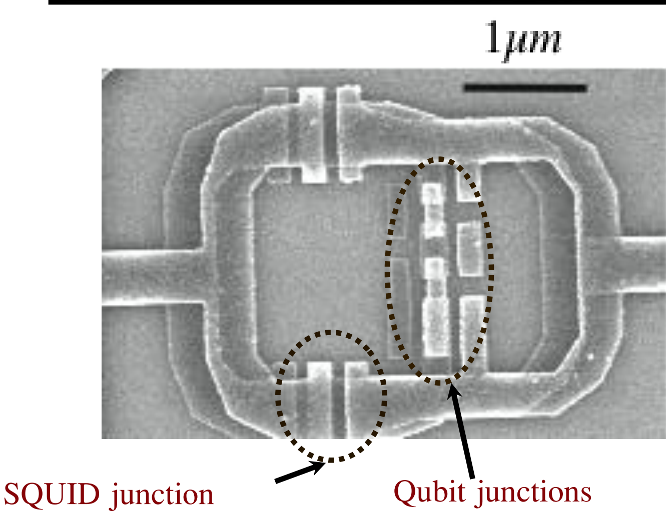


Tilted washboard potential of a Josephson junction for different values of a bias current

Examples of Josephson junctions: Persistent current qubits

Al





Il'ichev E. et al. Leibniz Institute of Photonic Technology, Germany

material: Aluminum,

shadow-evaporation technique,

two junctions 600x200nm

$I_C \approx$ 600 nA,

the third one is smaller:

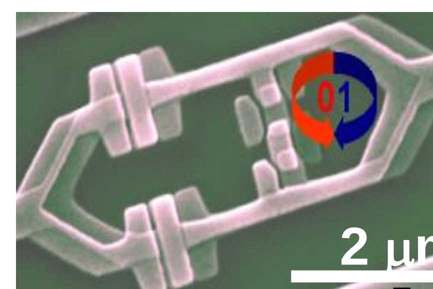
$a = E_{J1} / E_{J2,3} \sim 0.8 \ldots 0.9$,

inductance $L \approx$ 20-40 pH.

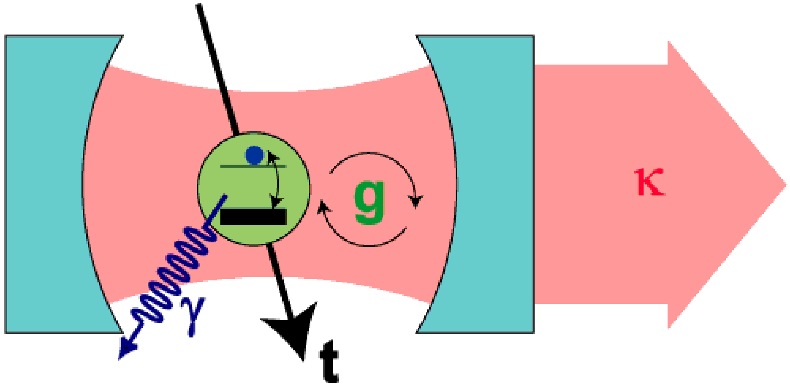
J.E. Mooij *et al.*, Science 285, 1036, 1999

flux qubit/Delft

$E_J/E_C \sim 40$



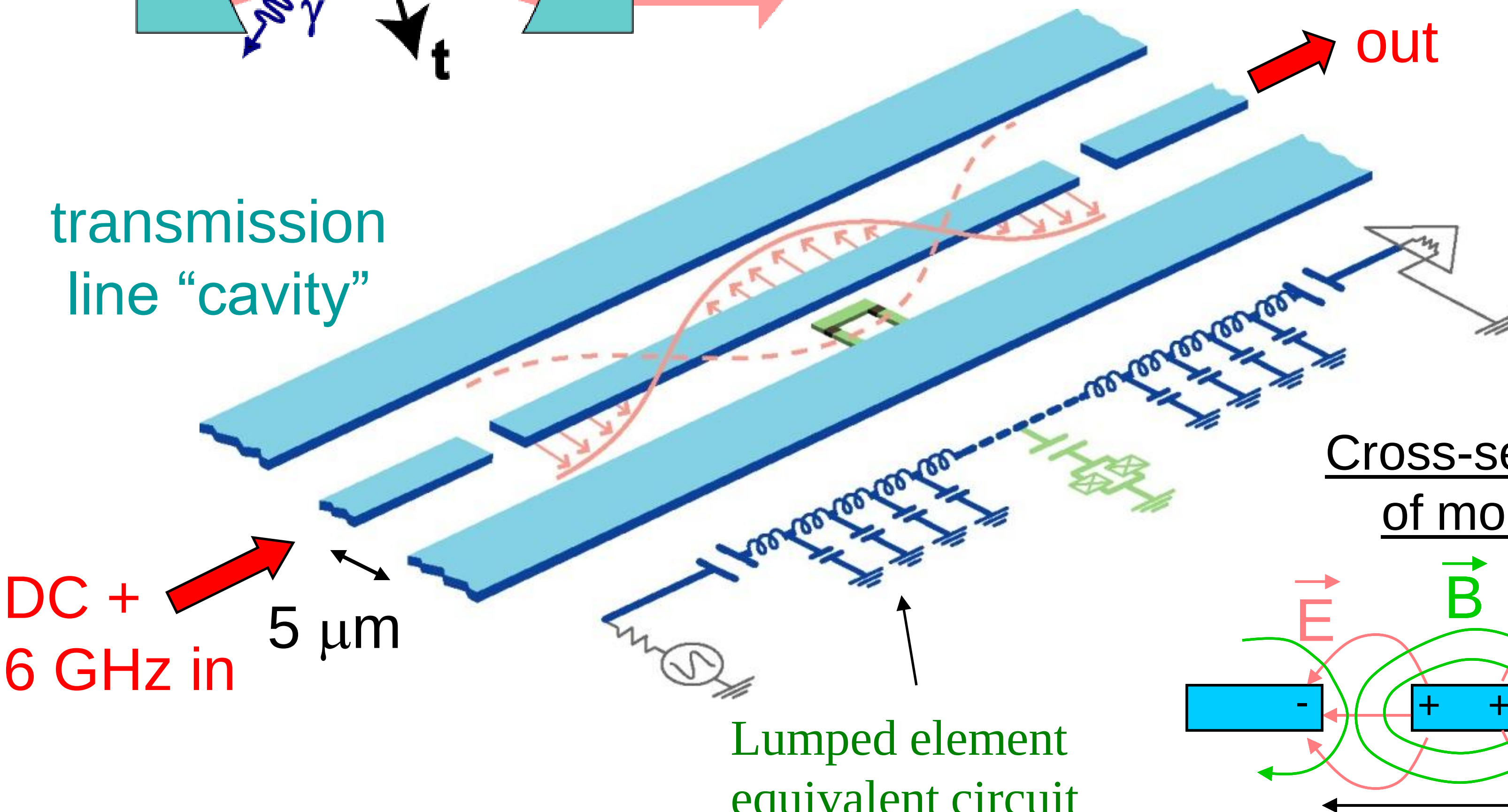
A circuit analog for cavity QED

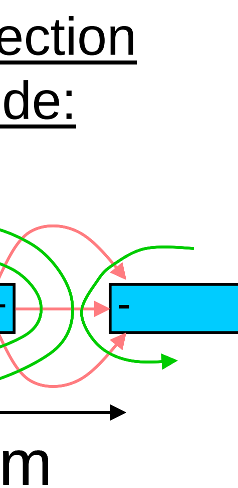


2g = vacuum Rabi freq.

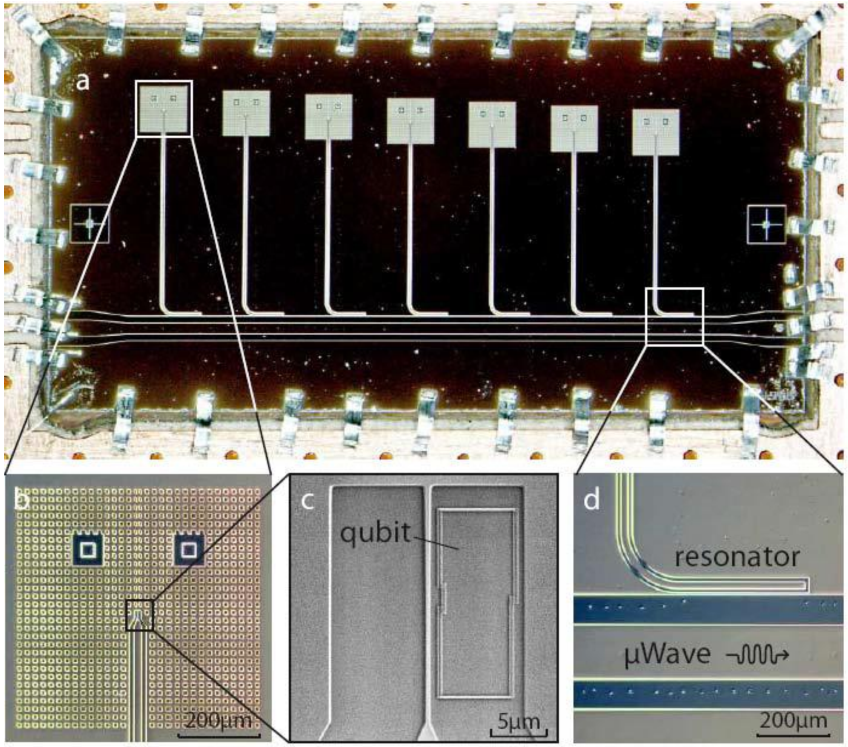
κ = cavity decay rate

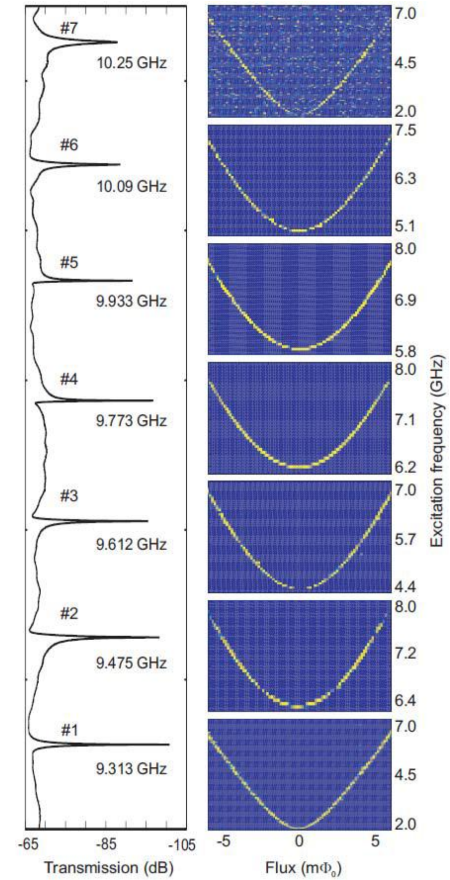
γ = "transverse" decay rate





Blais, Huang, Wallraff, SMG & RS, PRA 2004





M.Jerger, S.Poletto,P.Masha,
U.Hubner, F.Lukashenko,
E.Il'ichev, A.V.Ustinov (2011)

Implementation of a Quantum Metamaterial, Alexey V. Ustinov et al. (2013)

A. Shvetsov, A. M. Satanin, Franco Nori, S. Savel'ev, A.M. Zagoskin, Quantum metamaterial without local control, Phys. Rev. B 87, 235410(2013).

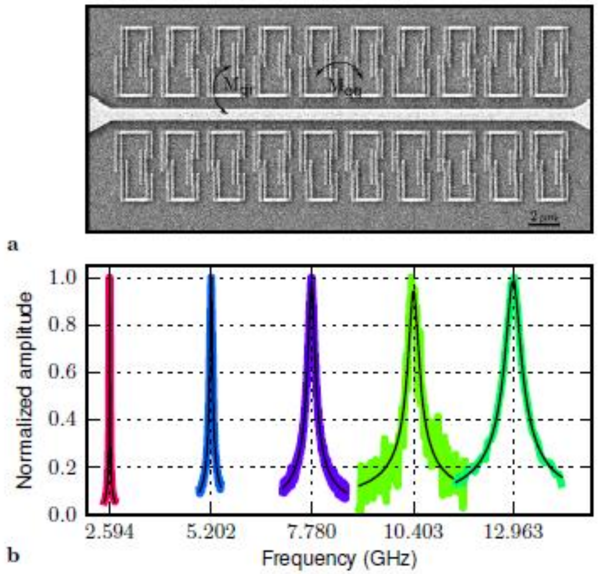
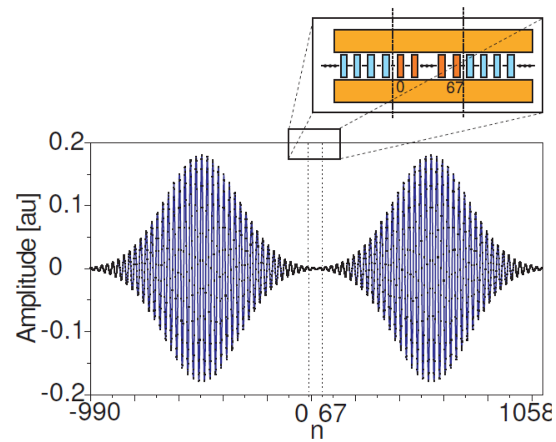
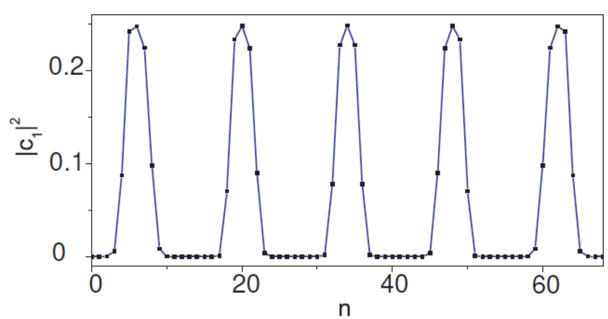
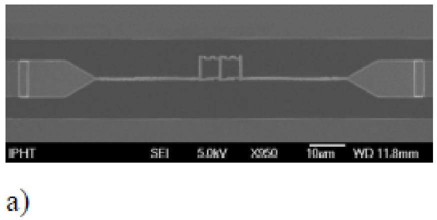
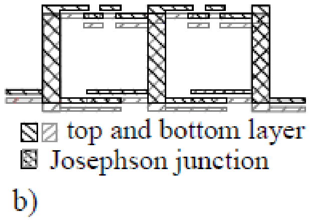


FIG. 1. **Micrograph of the sample and spectrum of the resonator.** **a,** Scanning electron micrograph of the sample









Two-photon lasing by a superconducting qubit, E. Il'ichev et al.(2015)

Superposition principle.

Is the Schrodinger's cat dead or alive?

No moon there

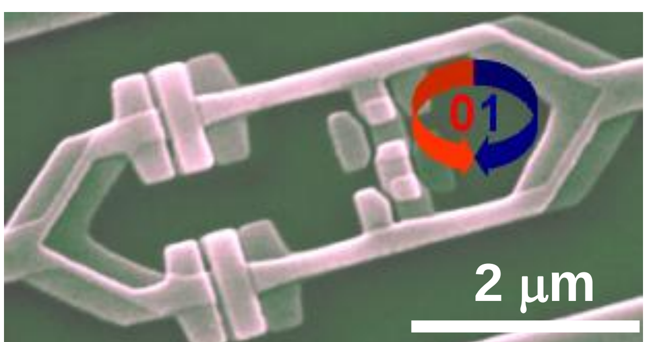
Johan E. Mooij

nature physics | VOL 6 | JUNE 2010,401

$$c_{\uparrow}\left|\uparrow\right\rangle + c_{\downarrow}\left|\downarrow\right\rangle$$

“I like to think that the moon is there even if I don’t look at it”, Albert Einstein once remarked. He objected to the notion that truly macroscopic objects might behave according to the laws of quantum mechanics, and thus be subject to the same uncertainties as photons or spins.

The moon — a small moon, admittedly — is not there!



Mooij *et al.*, Science 285, 1036, 1999

flux qubit/Delft $E_J/E_C \sim 40$

Entanglement

Entanglement is a term used in quntum theory to describe the way that particles of energy/matter can become ***correlated*** to predictably interact with each other regardless of how far apart they are.

E. Schrödinger, Naturwissenschaften 23, 807 (1935).

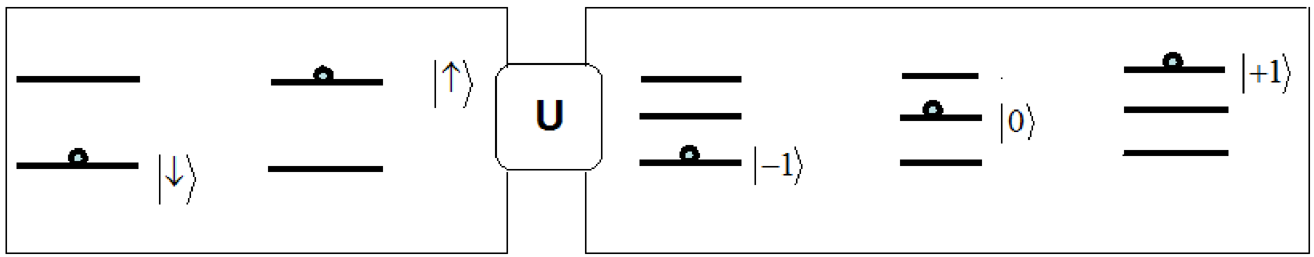
The superposition principle is the basis of quantum theory. When this principle is applied to composite systems a new concept of entanglement is appeared. It was introduced by Schrödinger in quantum theory in the last century and at present time this principle became a central topic in discussion as main resource of quantum information and quantum computational problems. Entanglement is a property shared by two or more correlated systems. Quantum correlations are also responsible for a number of interesting effects in mesoscopic systems. These correlations may be realized in superconducting waveguides and circuits with embedded Josephson junctions and such kind of circuits are considered as promising candidates for future quantum information processing.

A qubit is a two-state quantum-mechanical system,such as the polarization of a single photon:here the two states are vertical polarization and horizontal polarization.

A qutrit is a unit of quantum information that exists as a superposition of three orthogonal quantum states

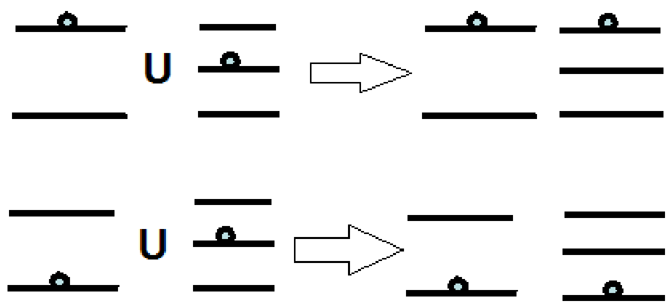
Interaction

«qubit» «qutrit»



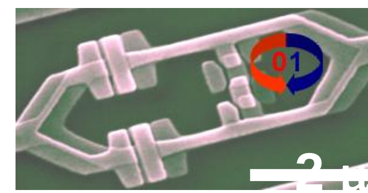
$\mathbf{U}|\uparrow,0\rangle = |\uparrow,+1\rangle$

$\mathbf{U}\,|\downarrow,0\rangle = |\downarrow,-1\rangle$



Superposition state of the qubit may be prepared by using of a Rabi pulse

$$c_\uparrow|\uparrow\rangle + c_\downarrow|\downarrow\rangle$$



An initial state of the decoupled system: $\left(c_\uparrow|\uparrow\rangle + c_\downarrow|\downarrow\rangle\right)\otimes|0\rangle = c_\uparrow|\uparrow,0\rangle + c_\downarrow|\downarrow,0\rangle$

$$\left(c_\uparrow|\uparrow\rangle + c_\downarrow|\downarrow\rangle\right)\otimes|0\rangle = c_\uparrow|\uparrow,0\rangle + c_\downarrow|\downarrow,0\rangle$$

Entanglement: $U\left(c_\uparrow|\uparrow,0\rangle + c_\downarrow|\downarrow,0\rangle\right) = c_\uparrow|\uparrow,+1\rangle + c_\downarrow|\downarrow,-1\rangle$

A maximal entangled state: $c_\uparrow = -c_\downarrow$ $\qquad |s\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow,+1\rangle - |\downarrow,-1\rangle\right)$

How to prepare entangled states of photons in the microwave frequency domain?

Selected for a Viewpoint in *Physics*
PRL **112,** 170501 (2014) PHYSICAL REVIEW LETTERS week ending 2 MAY 2014

Observation of Measurement-Induced Entanglement and Quantum Trajectories of Remote Superconducting Qubits

N. Roch,[1,*] M. E. Schwartz,[1] F. Motzoi,[2] C. Macklin,[1] R. Vijay,[3] A. W. Eddins,[1] A. N. Korotkov,[4] K. B. Whaley,[2] M. Sarovar,[5] and I. Siddiqi[1]

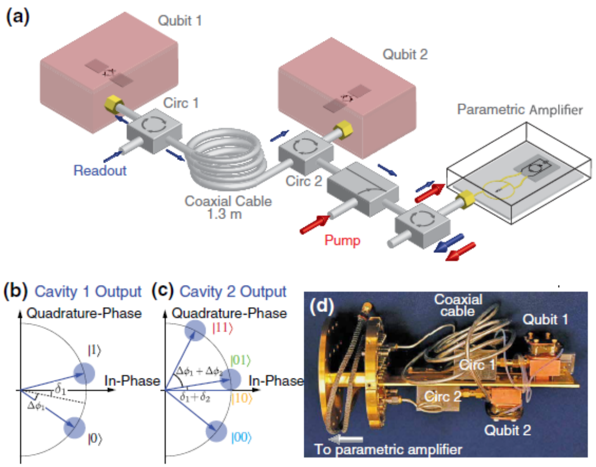


FIG. 1 (color online). Experimental setup. (a) Simplified representation of the experimental setup. (b) and (c) Schematic of the phase shift acquired by a coherent state sequentially measuring first qubit 1 (b) and then qubit 2 (c) in reflection. (d) Picture of the base-temperature setup.

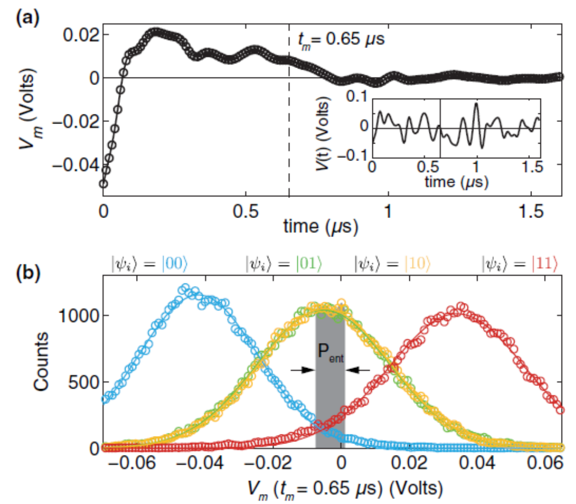
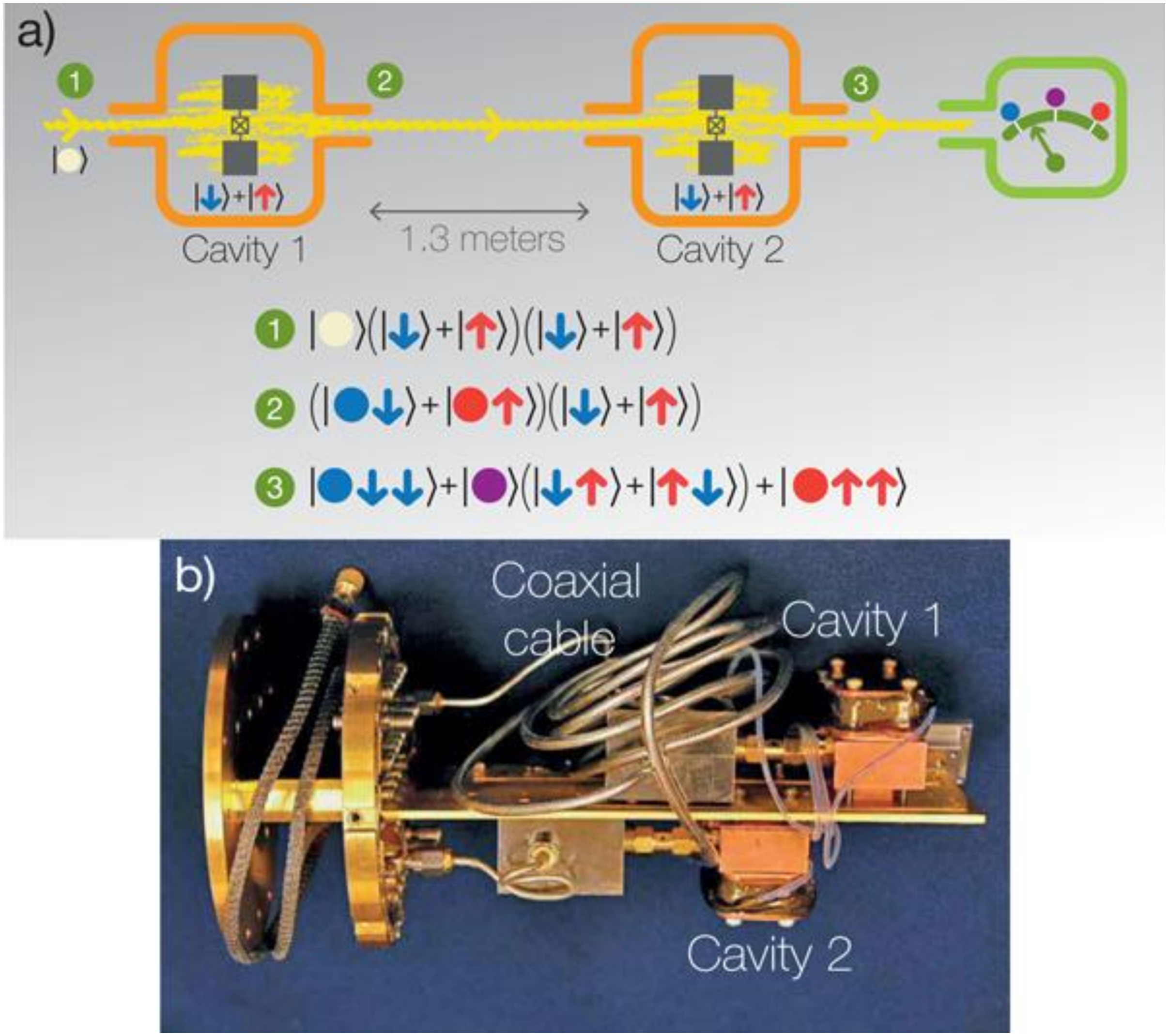
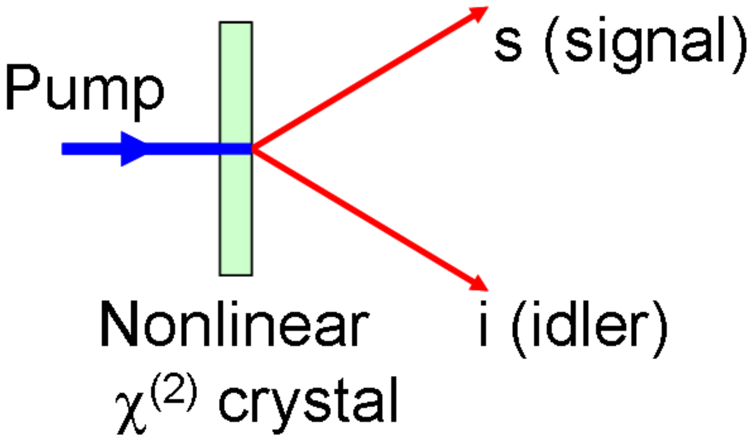


FIG. 2 (color online). Demonstration of indistinguishability between $|01\rangle$ and $|10\rangle$ computational states during measurement. (a) Example of the temporal evolution of the measurement signal V_m. The inset shows the associated instantaneous voltage $V(t)$.

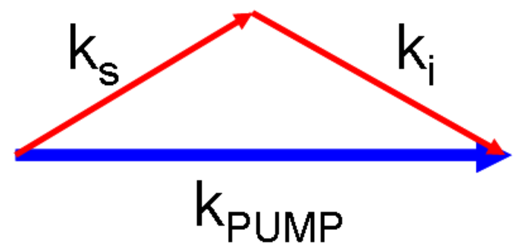

a)
1
2
3
Cavity 1
1.3 meters
Cavity 2
b)
Coaxial cable
Cavity 1
Cavity 2

Spontaneous parametric down-conversion in optics

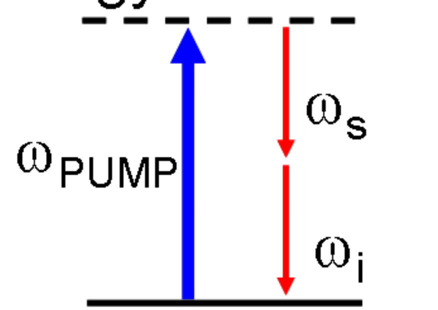
Spontaneous
Parametric
Downconversion



Momentum Conservation



Energy conservation

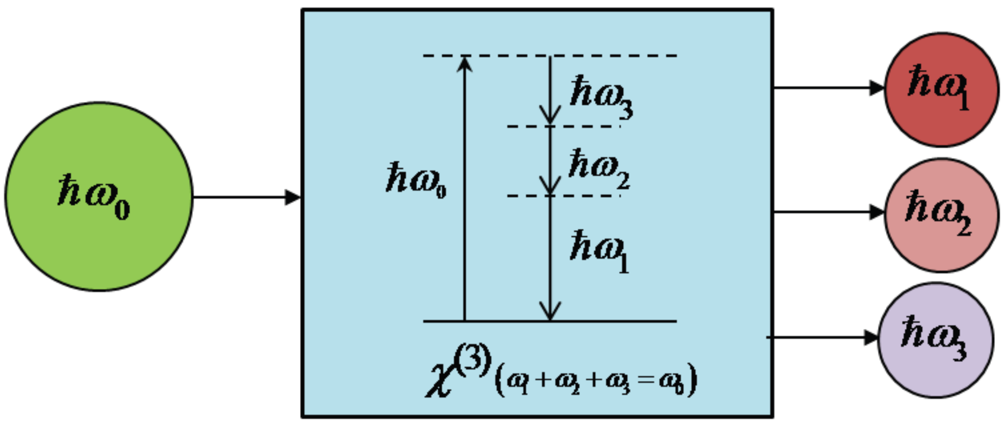


$$\varphi_{PUMP} = \varphi_s + \varphi_i$$

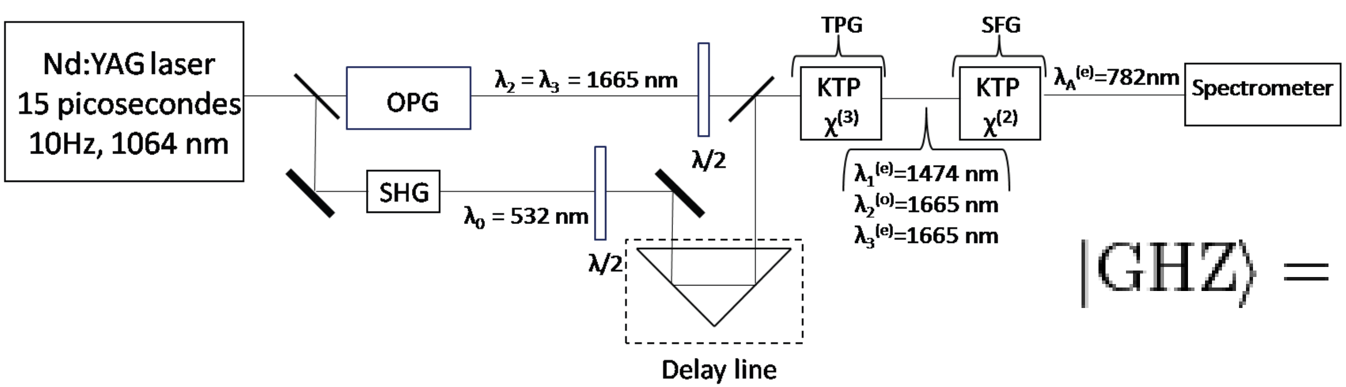
During this process governed by a third order electric susceptibility (3), three highly correlated photons, with the energies 1 , 2 and 3 , are created from the annihilation of a photon at 0 as shown in figure 1(a).

Indeed calculations showed that the simultaneous birth of three photons is at the origin of intrinsic three-body quantum properties such as three-particle Greenberger-Horne-Zeilinger (GHZ) quantum entanglement

Potassium Titanyl Phosphate (KTiOPO4 or KTP)



Greenberger-Horne-Zeilinger

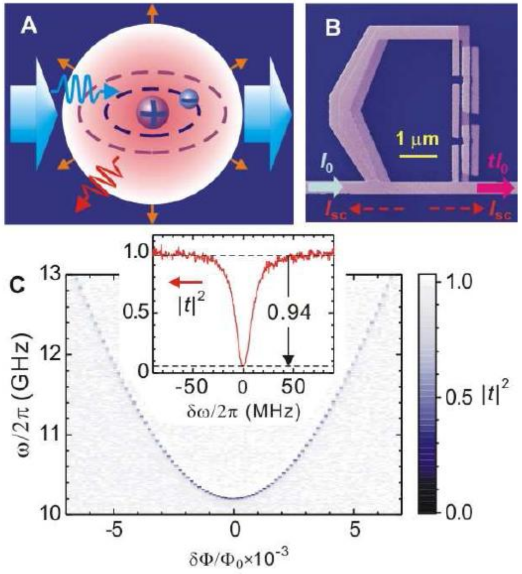


$$|\mathrm{GHZ}\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}}.$$

How to implement the microwave down-conversion effect in a waveguide with an Josephson junction?

Recently strong coupled systems such as waveguide and artificial atoms have attracted much attention.

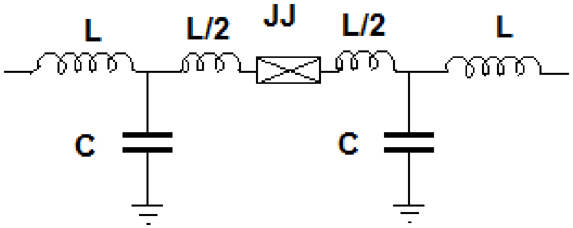
For instance, as first shown experimentally by Astafiev *et al.* (Science, 327, 840 (2010)) almost ideal mode matching can be realized with a superconducting flux qubit coupled to a 1D transmission line. In that experiment, 94% extinction of the transmitted signal was observed showing that a single qubit can act as a near ideal mirror for microwave light.



We study how a multi-level Josephson junction (artificial atom) interacts with an electromagnetic pulse in 1D coplanar waveguide. The main goal of this work is to study **nonlinear effects**. In particular, we discuss the down-conversion (the effect of dividing the frequency) in a waveguide with an embedded Josephson junctions

Transmission Lines. Telegraph equations

The equivalent circuit of the transmission line
 Lumped elements:
A capacitance and an inductance per unit length Josephson junctions

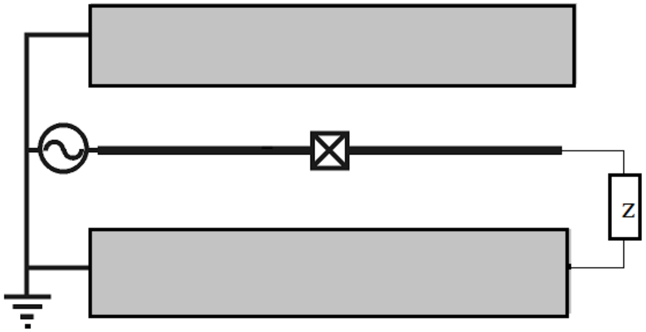


A transmission line, such as a coaxial cable or a co-planar waveguide can be approximated by a series of inductors with a parallel capacitance to ground

Kirchoff's laws

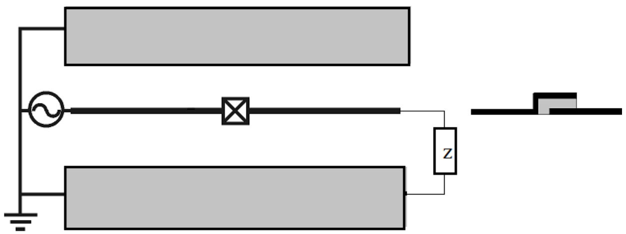
$$\begin{cases} V_{n+1} = V_n - \dfrac{L_l \Delta x}{c^2} \dfrac{\partial I_n}{\partial t} - V_J(t), \\ I_{n+1} = I_n - C_l \Delta x \dfrac{\partial V_n}{\partial t}, \end{cases}$$

Continuous limit: coplanar waveguide with an embedded Josephson junctions



$$\begin{cases} \dfrac{\partial V(x,t)}{\partial x} = -\dfrac{L_l}{c^2} \dfrac{\partial I(x,t)}{\partial t} - V_J(t)\delta(x), \\ \dfrac{\partial I(x,t)}{\partial x} = -C_l \dfrac{\partial V(x,t)}{\partial t}, \\ V_J(t) = \dfrac{\hbar}{2e} \dfrac{\partial \varphi(t)}{\partial t}, \end{cases}$$

Resonance modes in coplanar waveguide with integrated Josephson circuits. A classical system



$I_C = 5\cdot 10^{-7} A$

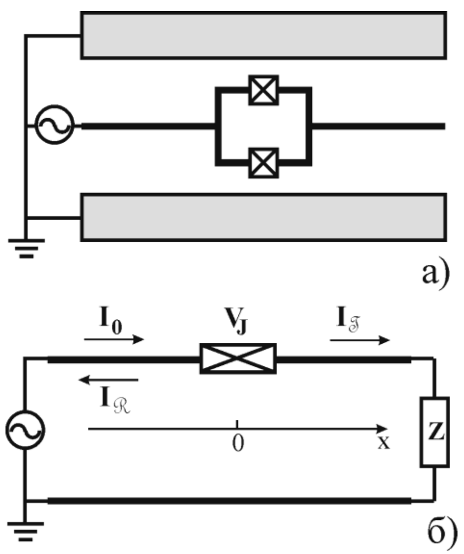
$C = 10^{-12}\Phi$

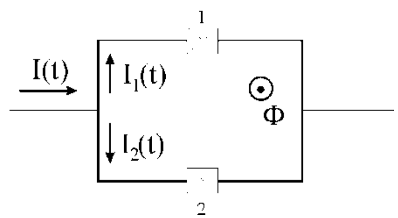
$R = 10^{4}\Omega$

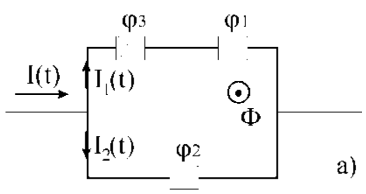
$Z_l = 18\Omega$

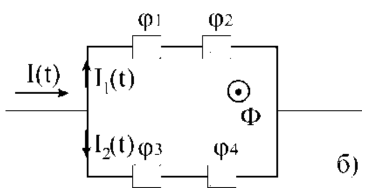
$\omega_{J,\max} = 5.5\cdot 10^{10} c^{-1}$

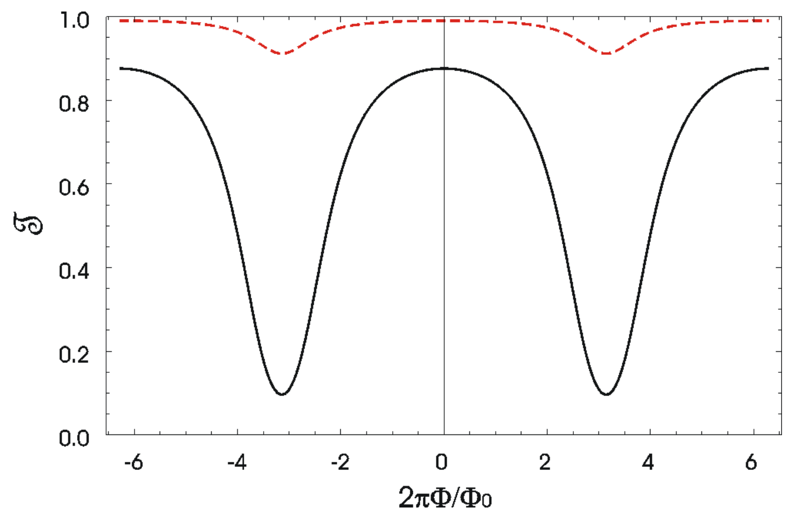
A.V.Shvetsov,A.M.Satanin,V.A.Mironov and E.Il'ichev,Low Temp. Phys. 39, 927 (2013)







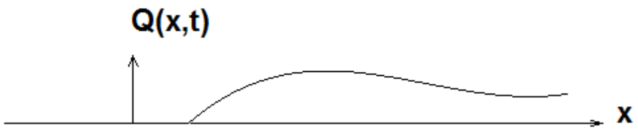




Current as a function of magnetic flux

Classical electrodynamics of waveguide

$$\begin{cases} \frac{\partial V(x,t)}{\partial x} = -\frac{L_l}{c^2}\frac{\partial I(x,t)}{\partial t} - V_J(t)\delta(x), \\ \frac{\partial I(x,t)}{\partial x} = -C_l\frac{\partial V(x,t)}{\partial t}, \\ V_J(t) = \frac{\hbar}{2e}\frac{\partial \varphi(t)}{\partial t}, \end{cases}$$



$$\begin{cases} I(x,t) = \frac{\partial Q(x,t)}{\partial t}, \\ V(x,t) = -\frac{\partial Q(x,t)}{C_l\ \partial x}. \end{cases}$$

$$\begin{cases} L_l\frac{\partial^2 Q(x,t)}{\partial t^2} - \frac{\partial^2 Q(x,t)}{C_l \partial x^2} = \frac{\hbar}{2e}\frac{\partial \varphi}{\partial t}\delta(x), \\ J\frac{\partial^2 \varphi}{\partial t^2} + J\omega_J^2 \sin\varphi = \frac{\hbar}{2e}\frac{\partial Q(0,t)}{\partial t}. \end{cases}$$

The Lagrangian of the electromagnetic field that interacts with the Josephson junction

$$\mathcal{L} = \int_{-\infty}^{\infty} \mathrm{dx}\left\{\frac{L_l}{2}\left(\frac{\partial Q(x,t)}{\partial t}\right)^2 - \frac{1}{2C_l}\left(\frac{\partial Q(x,t)}{\partial x}\right)^2 + \delta(x)\left(\frac{J}{2}\left(\frac{\partial \varphi}{\partial t}\right)^2 + J\omega_J^2 cos\varphi\right) - \delta(x)\frac{\hbar}{2e}\frac{\partial \varphi}{\partial t}Q(x,t)\right\}$$

The Hamiltonian of the system

$$\Phi(x,t) = L_l \frac{\partial Q(x,t)}{\partial t}$$

$$\mathcal{H} = \int_{-\infty}^{\infty} \left\{ \frac{\Phi(x,t)}{2L_l}^2 + \frac{1}{2C_l} \left(\frac{\partial Q(x,t)}{\partial x} \right)^2 + \left(\frac{1}{2J} \left(P + \frac{\hbar}{2e} Q(x,t) \right)^2 + J\omega_J^2 (1 - cos\varphi) \right) \delta(x) \right\}$$

We use the canonical quantization procedure which was developed by Heisenberg and Dirac

Variables $\Phi(x,t)$ and $Q(x,t)$ are canonically conjugate $\left[\hat{\Phi}(x,t), \hat{Q}(x',t)\right] = i\hbar\delta(x - x')$

$$\hat{Q}(x,t) = \frac{1}{\sqrt{L}} \sum_k \sqrt{\frac{\hbar}{2\omega(k)L_l}} \left(e^{ikx}\hat{b}_k(t) + e^{-ikx}\hat{b}_k^\dagger(t) \right)$$

$$\hat{\Phi}(x,t) = -i\frac{1}{\sqrt{L}} \sum_k \sqrt{\frac{\hbar\omega(k)}{2L_l}} \left(e^{ikx}\hat{b}_k(t) - e^{-ikx}\hat{b}_k^\dagger(t) \right)$$

$\hat{b}_k^\dagger(t)$ - creation and

$\hat{b}_k(t)$ - annihilation operators in Heisenberg picture

$$\left[\hat{b}_k(t), \hat{b}_{k'}^\dagger(t)\right] = \delta_{k,k'} \qquad \omega(k) = vk \qquad v = \frac{1}{\sqrt{C_l L_l}}.$$

L is the waveguide length

Below we will be interested only in the process of dividing the frequency. Restricting the weakly nonlinear regime excitation, we will keep only the terms $\sim \varphi^4$ corresponding to the expansion of the potential energy of the Josephson junction:

$$H_J = \frac{P^2}{2J} + \frac{J\omega_J^2\varphi^2}{2} - \frac{\mu\varphi^4}{4}, \quad \mu = J\omega_J^2/6$$

$$\hat{\varphi} = \sqrt{\frac{\hbar}{2J\omega_J}}(\hat{a} + \hat{a}^+), \quad \hat{P} = -i\sqrt{\frac{\hbar J\omega_J}{2}}(\hat{a} - \hat{a}^+),$$

The Hamiltonian operator

$$\hat{\mathcal{H}} = \sum_k \hbar\omega(k)\hat{b}_k^\dagger\hat{b}_k + \hbar\omega_J\hat{a}^\dagger\hat{a} - \frac{1}{4}\hbar\mu\left(\hat{a}^\dagger + \hat{a}\right)^4 + i\hbar g\left(\hat{a}^\dagger - \hat{a}\right)\hat{Q}(0,t) + \frac{1}{2J}\hat{Q}(0,t)^2$$

$$\hat{Q}(0,t) = \frac{1}{\sqrt{L}}\sum_k\sqrt{\frac{\hbar}{2\omega(k)L_l}}\left(\hat{b}_k(t) + \hat{b}_k^\dagger(t)\right)$$

$$g = \left(\frac{1}{2e}\right)\sqrt{\frac{\hbar\omega_J}{2J}}$$ is coupling parameter

Estimation

$$\hbar g\left(\hat{a}^{\dagger}-\hat{a}\right)\hat{Q}(0,t) \sim E_c\left(\frac{E_J}{E_c}\right)^{3/4}\left(\frac{I_{ac}}{I_c}\right) \qquad E_c=\frac{e^2}{2C} \qquad E_J=\frac{\hbar}{2e}I_c$$

$$\frac{1}{2J}\hat{Q}(0,t)^2 \sim E_J\left(\frac{I_{ac}}{I_c}\right)^2$$

If $E_J \sim E_c$ and $\frac{I_{ac}}{I_c} << 1$ we can neglect $\frac{1}{2J}\hat{Q}(0,t)^2$

Rotating wave approximation (RWA)

$$\hat{Q}(0,t)=\hat{Q}^{(+)}(0,t)+\hat{Q}^{(+)}(0,t)$$

$$\hat{Q}^{(-)}(0,t)=\frac{1}{\sqrt{L}}\sum_k\sqrt{\frac{\hbar}{2\omega(k)L_l}}\hat{b}_k^{\dagger}(t),$$

$$\hat{Q}^{(+)}(0,t)=\frac{1}{\sqrt{L}}\sum_k\sqrt{\frac{\hbar}{2\omega(k)L_l}}\hat{b}_k(t).$$

Keeping only the resonant terms, we rewrite expression of the Hamiltonian in the following form:

$$\hat{\mathcal{H}}_{\mathrm{r}}=\sum_k\hbar\omega(k)\hat{b}_k^{\dagger}\hat{b}_k+\hbar\omega_J\hat{a}^{\dagger}\hat{a}-\frac{1}{4}\hbar\mu\left(\hat{a}^{\dagger}+\hat{a}\right)^4+i\hbar g\left(\hat{a}^{\dagger}\hat{Q}^{(+)}(0,t)-\hat{a}\hat{Q}^{(-)}(0,t)\right)$$

$$\frac{\partial \hat{b}_k}{\partial t} = \frac{1}{i\hbar}\left[\hat{b}_k, \hat{\mathcal{H}}_r\right] \qquad \frac{\partial \hat{a}}{\partial t} = \frac{1}{i\hbar}\left[\hat{a}, \hat{\mathcal{H}}_r\right]$$

$$\begin{cases} \dfrac{\partial \hat{b}_k}{\partial t} = -i\omega(k)\hat{b}_k - g\sqrt{\dfrac{\hbar}{2L\omega(k)L_l}}\hat{a}, & (b) \\ \dfrac{\partial \hat{a}}{\partial t} = -i\omega_J \hat{a} - \mu\left(\hat{a}^\dagger + \hat{a}\right)^3 + g\hat{Q}^{(+)}(0,t). & (a) \end{cases}$$

The main resonance (1:1)

For a weak driving pulse, a single Josephson junction operates as a linear oscillator and demonstrates the linear response for an excitation. At the same time a strong driving pulses can cause the transition of a Josephson oscillator in a nonlinear regime of excitation.

$$\hat{b}_k(t) = \hat{b}_k(0)e^{-i\omega(k)t} - g\sqrt{\frac{\hbar}{2L\omega(k)L_l}}\, e^{-i\omega(k)t}\int_0^t dt' e^{i\omega(k)t'}\hat{a}(t')$$

$$\frac{\partial \hat{a}}{\partial t} + i\omega_J \hat{a} + \mu\left(\hat{a}^\dagger + \hat{a}\right)^3$$

$$+g^2\sum_k \frac{\hbar}{2L\omega(k)L_l}\int_0^t dt' e^{i\omega(k)(t'-t)}\hat{a}(t') = g\hat{Q}^{(+)}_{ext}(0,t)$$

Where $\hat{Q}^{(+)}_{ext}(t - x/v)$

is the solution

$$\frac{\partial^2 \hat{Q}^{(+)}_{ext}(t-x/v)}{v^2\partial t^2} - \frac{\partial^2 \hat{Q}^{(+)}_{ext}(t-x/v)}{\partial x^2} = 0$$

and can be considered as an operator of incoming waves

A slow operator $\hat{a}_s(t)$

$$\hat{a}_s(t) = \hat{a}(t)e^{i\omega_J t}$$

$$g^2\sum_k \frac{\hbar}{2L\omega(k)L_l} e^{i\omega_J t}\int_0^t dt' e^{i\omega(k)(t'-t)}e^{-i\omega_J t}\hat{a}_s(t') = \left(g^2\sum_k \frac{\hbar}{2L\omega(k)L_l} e^{i\omega_J t}\int_0^t dt' e^{i\omega(k)(t'-t)}e^{-i\omega_J t'}\right)\hat{a}_s(t)$$

$$g^2\sum_k \frac{\hbar}{2L\omega(k)L_l}\int_0^t dt' e^{i(\omega(k)-\omega_J)(t'-t)} = g^2\frac{\hbar}{L_l}\int_{\omega_{\min}}^{\infty}\frac{d\omega}{\omega}\int_0^t d\tau e^{i(\omega_J-\omega)\tau} = i\delta\omega_L - \gamma$$

$$\delta\omega_L = g^2\frac{\hbar}{2eL_l}\int_{\omega_{\min}}^{\infty}\frac{d\omega}{\omega}P\left(\frac{1}{\omega_J-\omega}\right)$$ Lamb shift

$$\gamma = g^2\frac{\hbar}{2eL_l\omega_J}$$ decay term ("friction")

$$i\frac{\partial\hat{a}_s}{\partial t} = \left(\delta\omega_J - i\gamma - 3\mu\right)\hat{a}_s - 3\mu\left(\hat{a}_s^\dagger\hat{a}_s\right)\hat{a}_s + g\left(e^{i\omega_J t}\hat{Q}_{ext}^{(+)}(0,t)\right)_{aver}$$

$$\gamma = 0$$

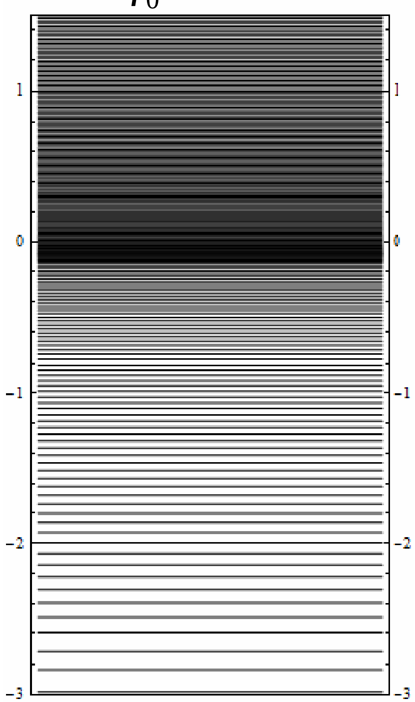
$$i\frac{\partial \hat{a}}{\partial t} = (\omega_J - 3\mu)\hat{a} \; - 3\mu\left(\hat{a}^{\dagger}\hat{a}\right)\hat{a} \; + f_0,$$

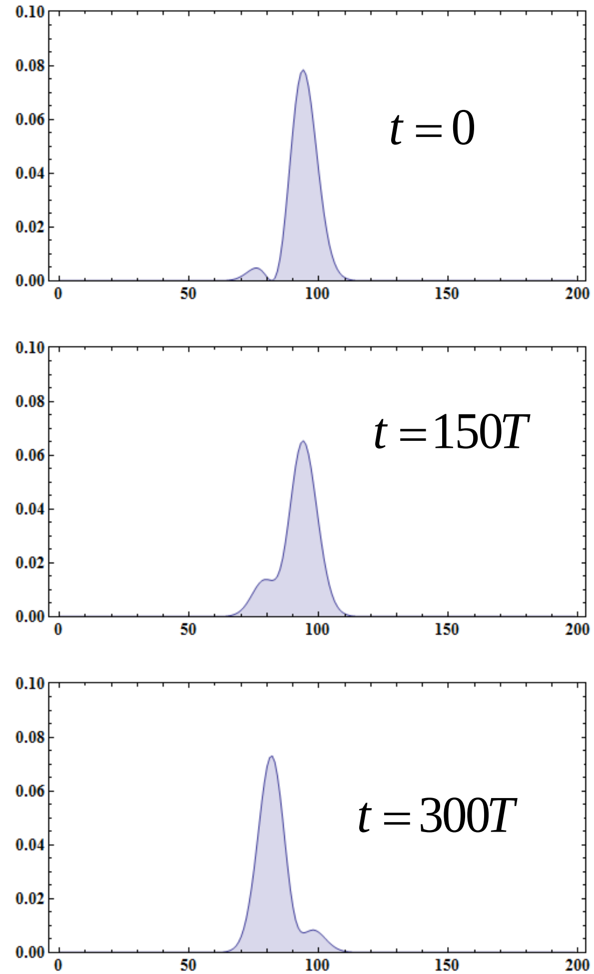
$$f_0 = g\left(e^{i\omega_J t}\hat{Q}^{(+)}_{ext}(0,t)\right)_{aver}$$

$$|\Psi\rangle = \sum C_n(t)|n\rangle$$

quasienergy states

$$f_0 = 0.05$$





The evolution of the superposition of quasi-energy states

Quantum theory of microwave down conversion

The waveguide is fed from the source of the microwave field, which can be prepared either in the Fock or in a coherent state $\hat{Q}_{ext}(t - x/v)$

Apply linear shift:

$$\begin{cases} \hat{Q}(x,t) = \hat{Q}_0(x,t) + \hat{q}(x,t), \\ \hat{\Phi}(x,t) = \hat{\Phi}_0(x,t) + \hat{\phi}(x,t). \end{cases} \qquad \begin{cases} \hat{P} = \hat{P}_0 + \hat{\Pi}, \\ \hat{\varphi} = \hat{\varphi}_0 + \hat{\psi}. \end{cases}$$

$$\hat{q}(x,t) = \frac{1}{\sqrt{L}} \sum_k \sqrt{\frac{\hbar}{2\omega(k) L_l}} \left(e^{ikx} \hat{b}_k(t) + e^{-ikx} \hat{b}_k^\dagger(t) \right)$$

$$\begin{cases} \dot{\hat{Q}}_0 = \dfrac{\hat{\Phi}_0}{L_l}, \\ \dot{\hat{\Phi}}_0 = \dfrac{1}{C_l} \dfrac{\partial^2 \hat{Q}_0(x,t)}{\partial x^2} - \dfrac{\hbar}{2eJ} \left(\hat{P}_0 + \dfrac{\hbar}{2e} \hat{Q}_0(x,t) \right) \delta(x), \\ \dot{\hat{\varphi}}_0 = \dfrac{1}{J} \left(\hat{P}_0 + \dfrac{\hbar}{2e} \hat{Q}_0(x,t) \right), \\ \dot{\hat{P}}_0 = -J \omega_J^2 \hat{\varphi}_0. \end{cases}$$

the auxiliary fields

$$\hat{\mathcal{H}} = \sum_k \hbar\omega(k) \hat{b}_k^\dagger \hat{b}_k + \hbar\omega_J \hat{a}^\dagger \hat{a} - \frac{1}{4} \hbar\mu \left(\hat{a}^\dagger + \hat{a} + \xi\varphi_0(t) \right)^4 + i\hbar g \left(\hat{a}^\dagger - \hat{a} \right) \hat{q}(0,t) + \frac{1}{2J} \hat{q}(0,t)^2$$

$$\begin{cases} \dfrac{\partial \hat{b}_k}{\partial t} = -i\omega(k)\hat{b}_k - g\sqrt{\dfrac{\hbar}{2L\omega(k)L_l}}\,\hat{a}, & (b) \\ \dfrac{\partial \hat{a}}{\partial t} = -i\omega_J \hat{a} - \mu\left(\hat{a}^\dagger + \hat{a} + \xi\varphi_0(t)\right)^3 + g\hat{q}^{(+)}(0,t). & (a) \end{cases}$$

$$\frac{\partial^2\hat{\varphi}_0}{\partial t^2}+\gamma\frac{\partial\hat{\varphi}_0(t)}{\partial t}+\omega_J^2\hat{\varphi}_0=\frac{\hbar}{2eJ}\hat{I}_{ext}(0,t),\quad \hat{I}_{ext}(0,t)=-i\frac{1}{\sqrt{L}}\sum_k\sqrt{\frac{\hbar\omega(k)}{2L_l}}\left(\hat{b}_k(0)e^{-i\omega(k)t}-\hat{b}_k^\dagger(0)e^{i\omega(k)t}\right)$$

$$\hat{\varphi}_0(t)=\frac{1}{\sqrt{L}}\sum_k\left(A_k\hat{b}_k(0)e^{-i\omega(k)t}+B_k\hat{b}_k^\dagger(0)e^{i\omega(k)t}\right)$$

$$\frac{\partial\hat{a}(t)}{\partial t}+i\omega_J\hat{a}(t)+\mu\left(\hat{a}^\dagger(t)+\hat{a}(t)+\xi\hat{\varphi}_0(t)\right)^3$$

$$+g^2\sum_k\frac{\hbar}{2L\omega(k)L_l}\int_0^t dt'e^{i\omega(k)(t'-t)}\hat{a}(t')=g\hat{q}_{ext}^{(+)}(0,t)$$

Let the characteristic frequency of the operator $Q_{ext}(0,t)=Q_m\cos(3\omega t),\quad \omega\sim\omega_J$

Quantum theory of fractional resonance

A linear shift

$$\begin{cases} P = \Pi + P_0, \\ \varphi = \phi + \varphi_0 \end{cases} \qquad \begin{cases} \dot{\varphi}_0 = P_0 / J, \\ \dot{P}_0 = -J\omega_J^2 \varphi_0 + f(t). \end{cases}$$

$$H = \frac{\Pi^2}{2J} + \frac{J\omega_J^2 \phi^2}{2} - \frac{\beta\left(\varphi_0 + \phi\right)^4}{4}$$

$$H = \omega_p \hat{a}^+ \hat{a} - \frac{\tilde{\mu}}{4}(\hat{a} + \hat{a}^+ + \alpha(t) + \alpha^*(t))^4,$$

$$\alpha(t) = i\, f_0 \int_0^t e^{i\omega\tau} \cos(3\omega\tau)\, d\tau$$

After the transition to a rotating coordinate system and averaging of rapidly oscillating oscillations, we obtain

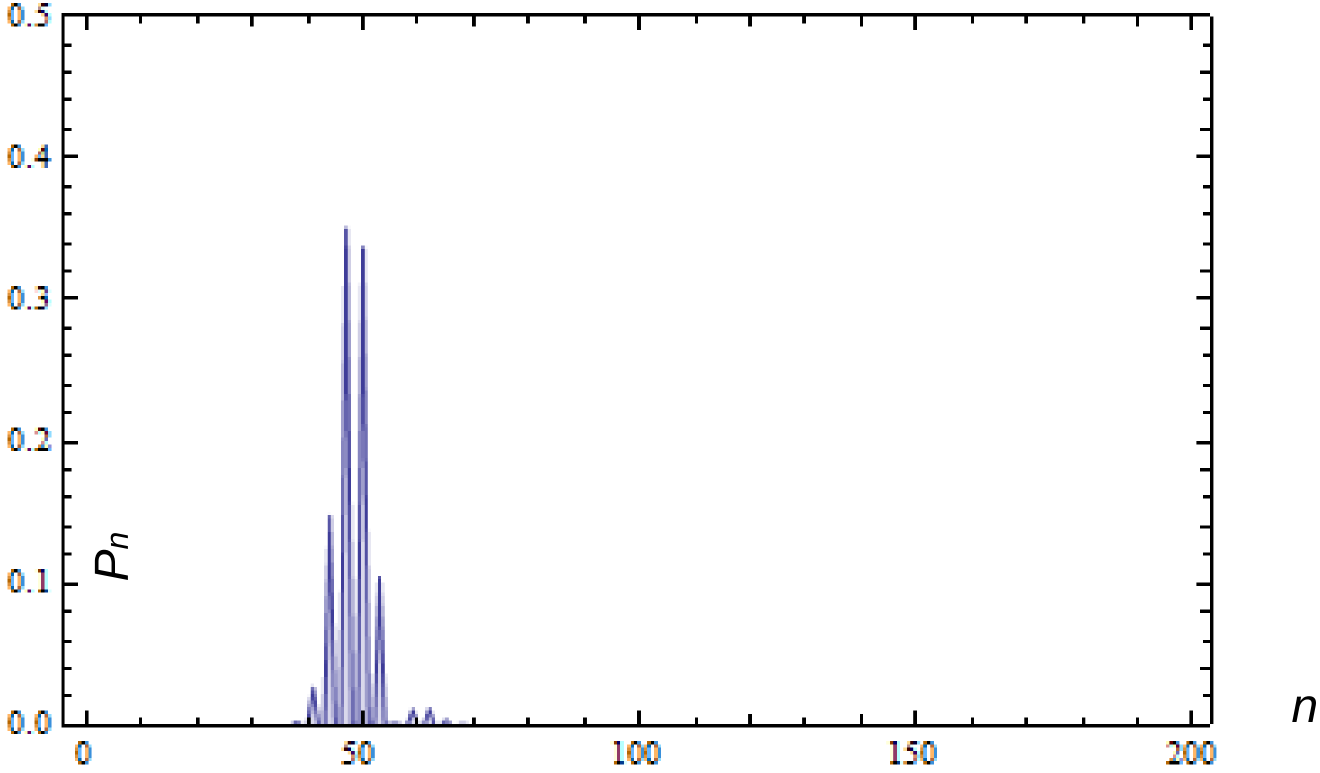
$$H_{eff} = \delta\hat{n} - \frac{\tilde{\mu}}{4}\hat{n}^2 - g(\hat{a}^3 + \hat{a}^{+3}) \qquad g = \frac{f_0 \omega_J}{8\sqrt{2}}$$

Schrödinger equation is solved in the Fock basis:

$$i\frac{\partial}{\partial t}|\psi> = \left(\delta\hat{n} - \frac{\tilde{\mu}}{4}\hat{n}^2 - g(\hat{a}^3 + \hat{a}^{+3})\right)|\psi>$$

$$|\psi> = \sum C_n(t)\left|n\right\rangle$$

Population dynamics of fractional resonance



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Results

1. Let there to be an incoming electromagnetic pulse in syperposition of coherent state in a waveguide with a narrow spectrum and with a characteristic frequency ~3ω; $\omega_J \sim \omega$

2. The pulse excites a nonlinear oscillator. The nonlinear oscillator is captured into a nonlinear (1:3) resonance:

$$i\frac{\partial \hat{a}}{\partial t} = \left(\omega - \omega_J - i\gamma - 3\mu\right)\hat{a} - 3\mu\left(\hat{a}^{\dagger}(t) + \hat{a}(t) + \xi\hat{\varphi}_0(t)\right)^3 = g\hat{q}_{ext}^{(+)}(0,t)$$

3. The oscillator emits an electromagnetic field at a frequency $\omega_J \sim \omega$

.

The shape of line is approximately Lorentzian when the oscillator is at the high excitation levels.

Resonant approximation for field-Josephson junction

$$\hat{\mathcal{H}} = \sum_{k} \hbar\omega(k)\hat{b}_k^{\dagger}\hat{b}_k + \hbar\omega_J \hat{a}^{\dagger}\hat{a} \; -$$

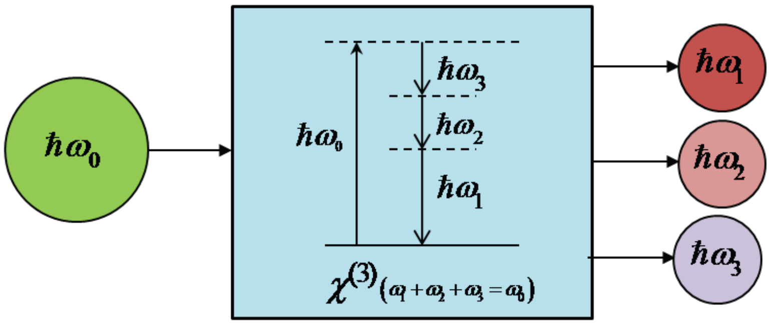
$$-\frac{\hbar^2\beta}{4\left(2J\omega_J\right)^2}\left(\hat{a}\,e^{-i\omega_J t} + \hat{a}^{\dagger}e^{i\omega_J t} + \xi\sum_k \sqrt{\frac{\omega_J \hbar L_l}{L\omega(k)}}\left(A_k e^{-i\omega(k)t}\hat{b}_k + B_k e^{i\omega(k)t}\hat{b}_k^{\dagger}\right)\right)^4 \; .$$

$$k_0 \approx 3\omega_J / v \qquad \omega(k_0) \approx 3\omega_J \qquad b_{k_0}^{\dagger}\left|0\right\rangle$$

$$\left|\Psi(t)\right\rangle = \sum C_k^{(1)}(t)\, b_k^{\dagger}\left|0\right\rangle + \sum C_{k_1k_2k_3}^{(3)}(t)\, b_{k_1}^{\dagger} b_{k_2}^{\dagger} b_{k_3}^{\dagger}\left|0\right\rangle$$

The one-photon amplitude $C_k^{(1)}(t)$

and $C_{k_1k_2k_3}^{(3)}$ - three-photon amplitude



Summary

- Developed: a simple theory of microwave quantum field in a waveguide with an integrated Josephson junction.

- Shown: in the case of coherent excitation the nonlinear oscillator can be captured by an external force into either main or fractional resonances.

- Constructed: under the rotating wave approximation, the quasi-energy states of an effective Hamiltonians describing the main and fractional resonances.

- Predicted: in the nonlinear Josephson circuit, it is possible to observe the down-conversion effect.