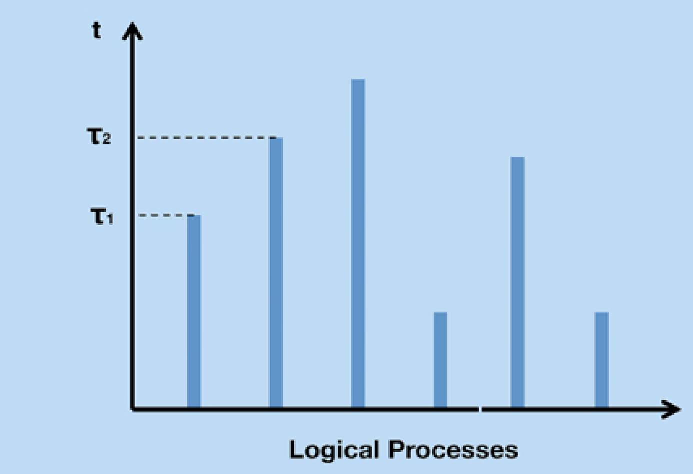
Virtual Time Profile Modeling in Parallel Discrete Event Simulation

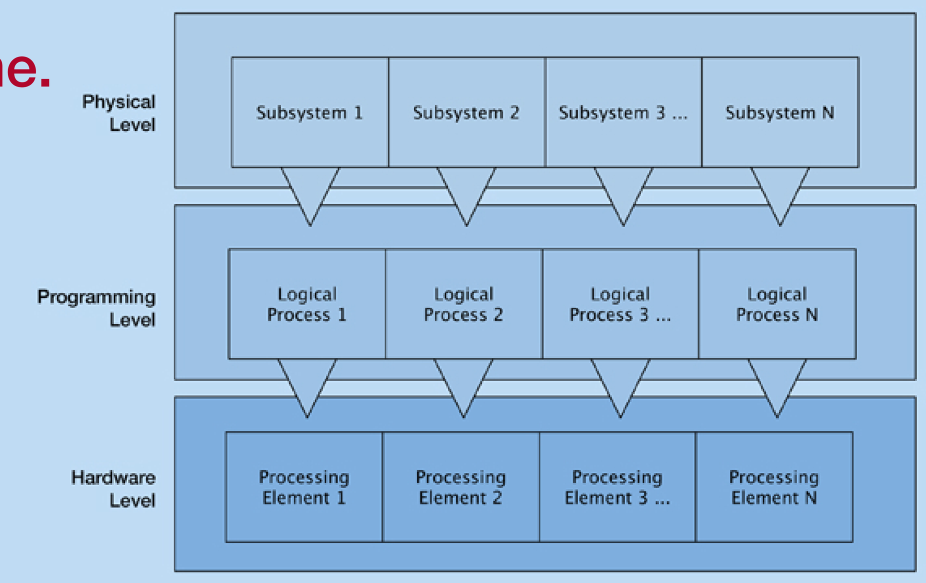
Authors: Liliya Ziganurova and Shchur L.N.
Science Center in Chernogolovka, 142432, Chernogolovka
National Research University “Higher School of Economics”, 101000, Moscow

1. What is PDES?

Parallel Discrete Event Simulation is a space-parallel approach with an execution of a single discrete event simulation program on a parallel computer or on a cluster of computers [1].

The essentials of PDES:
1) Changes of subsystems occure at some instants of time and are called discrete events.
2) If some objects have dependences we need to preserve causality using some synchronization protocol.
3) Using message sending mechanism.
4) No information exchange and no shared memory between subsystems.
5) The concept of Virtual Time.





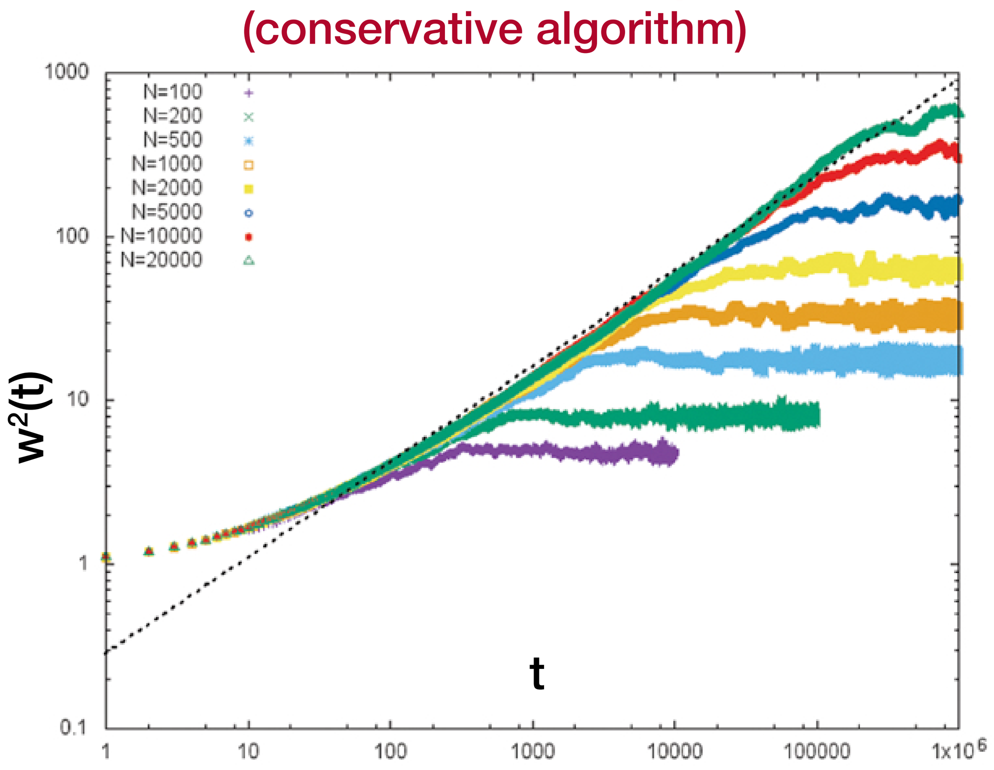
2. Time Evolution

A Local Virtual Time (LVT) τ_i is associated with each object *i*. The event generated by an object *i* is stamped with the LVT τ_i. Each LP develops in time changing its internal state and its LVT.

Time of simulation is measured in the Global Virtual Time (GVT) which is a minimum of the LVT profile [2]. It is important each LP receive messages with the time stamp less than its LVT otherwise causality is violated.

Fujimoto classified PDES algorithms into two groups: optimistic algorithms and conservative algorithms [1,3]. There is also the third group which may be presented by FaS algorithm [4].

1. Evolution of time profile
(conservative algorithm)



2. Parameters of fit funtion for u(t)
(optimistic algorithm)

Integer τ				Real τ			
F	U_0	q_c	a	F	U_0	q_c	a
1	1.47(6)	0.229(1)	1.73(3)	1	1.08(2)	0.149(2)	1.63(2)
2	1.30(3)	0.1708(8)	1.72(2)	2	1.06(2)	0.151(2)	1.60(2)
4	1.29(2)	0.1455(6)	1.78(1)	4	1.12(2)	0.147(2)	1.65(2)
6	1.32(2)	0.1382(7)	1.82(1)	6	1.08(2)	0.149(2)	1.62(2)
8	1.32(3)	0.1354(8)	1.83(2)	8	1.07(2)	0.149(2)	1.62(2)
10	1.35(3)	0.1335(7)	1.85(2)	10	1.07(2)	0.150(2)	1.61(2)

3. Conservative Algorithm

Subsystem waits for all events to happen in every subsystem from each it depends on, before proceed in time. Therefore, LVT τ_i of LP(i) should not be greater than LVT τ_k of all LPs from which LP*(i)* depends on [1].

The simplest case of application of conservative algorithm is to the chain of LPs communicating with neighbors only. Evolution of LVT profile may be written as iterative process:

$\tau_i(t+1) = \tau_i(t) + \eta_i(t)$ *if* $\tau_i(t) \le \min\{\tau_i-1(t), \tau_i+1(t)\}$, *and* $\tau_i(t+1) = \tau_i(t)$, *otherwise.*
$\eta_i(t)$ are random variables.

Numerical simulation of Korniss et al model gives average speed of the time horizon is 0.246410(7) [4]. It means that **this algorithm is free of deadlock.**

In the continuum limit the LVT time profile obeys the equation introduced by Kardar-Parisi-Zhang (KPZ) for the surface growth on the solid substrate [4,5]. Mapping this result from KPZ onto the properties of LVT time profile in PDES shows us that width of LVT grows with time as $t^{2\beta}$ and with numbers of LPs as $t^{2\alpha}$, $\alpha=1/2$, $\beta=1/3$. This means that **LPs are unsynchronized with the simulation time growth.**

My numerical results: *avg speed=0.24644(3),* $\alpha=0.4(3)$, $\beta=0.2915(1)$

4. Optimistic Algorithm

All LPs are evaluated in time with assumption that causality is fulfilled. An optimistic algorithm introduces a protocol of rollback in time that resolves the problem of causality. The most well known optimistic protocol is **Time Warp** [6].

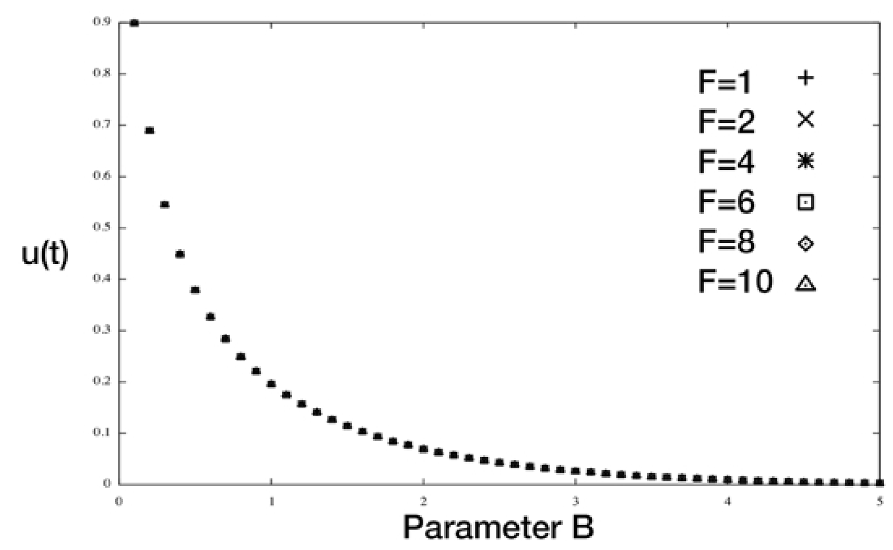
2 steps of optimistic mechanism:
1) unrestricted evolution of LVT **forward** in time (parameter *F*).
2) rollback (or **backward**) algorithm of sending anti-messages (parameter *B*).

It was found in [7] that the average value of the speed profile *u(t)* evolves as $U(t,q) = U_0(q - q_c)^a$, with $q=1/(1+B)$, with a value of $a=1.74$ close to the critical exponent of directed percolation $a\approx1.73$ [8]. The value of $q_c=0.23(3)$.

In my numerical simulation in case of integer τ which depends on F, I obtained for $F=1$ $q_c=0.229(1)$ and $a=1.73(3)$. But the universality was not observed.

With real τ the results for different parameters F are equal within the error. In this case $q_c=0.149(2)$ and $a=1.63(2)$ (see Figure 2,3).

3. The speed of profile *u(t)*
(optimistic algorithm)



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Contacts: ziganurova@gmail.com