Notice
on
REDACTIONS

Due to the timely nature of the presented slides, some have been redacted by eliminating complete slides. The redaction has been performed:

- ***To protect proprietary information***
- ***To potentially protect from copywriting infringement***
- ***To not allow any journal to claim that the new results presented had already been published, thereby eliminating double-publication appearances.***

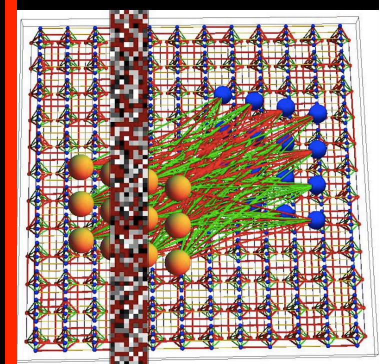
The authors apologize for these legal impediments to advancing science.

Spanning Trees, Continents, and the Quantum/Classical Divide on D-Wave 2 Machines

Mark A. Novotny

CSP2015
International Conference on Computer Simulation in Physics and beyond

National Research University Higher School of Economics
Moscow, Russia
September 6-10, 2015



MISSISSIPPI STATE UNIVERSITY

Spanning Continents D-Wave 2 Machines

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Jülich Supercomputing Centre
RWTH Aachen U

Spanning Continents
Dollars and Euros

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J. Spencer Hall

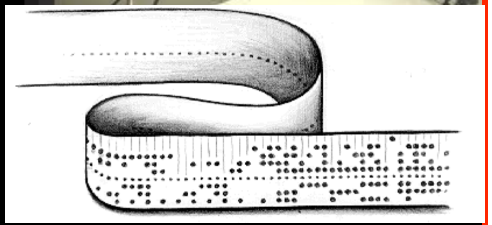
Lukas Hobl
Kristel Michielsen

NSF DMR-1206233

Jülich Supercomputing Centre

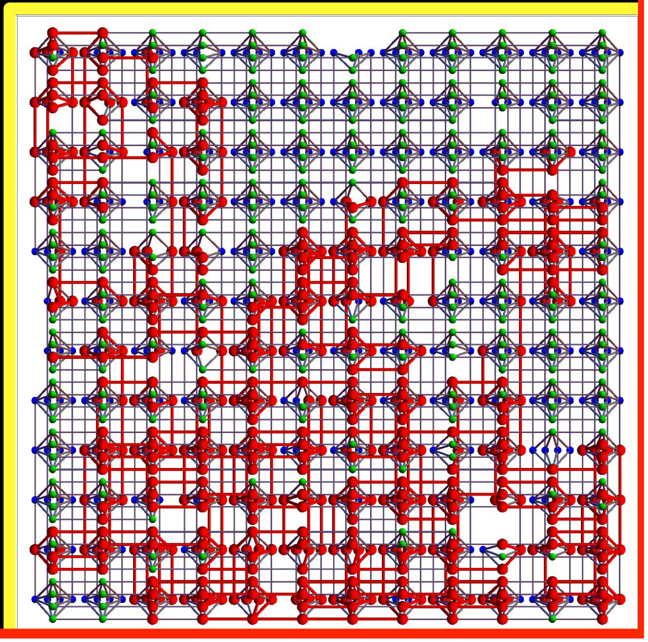
Who has done computational physics …

Computer CPU based on transistors?



Data storage on paper tape?

RAM from coil-wound solenoids?



Programmed in Assembly language?

A 1000+ qubit quantum computer?

Disruptive to computing

Been doing computational physics since
PhD in Physics from Stanford University in 1978

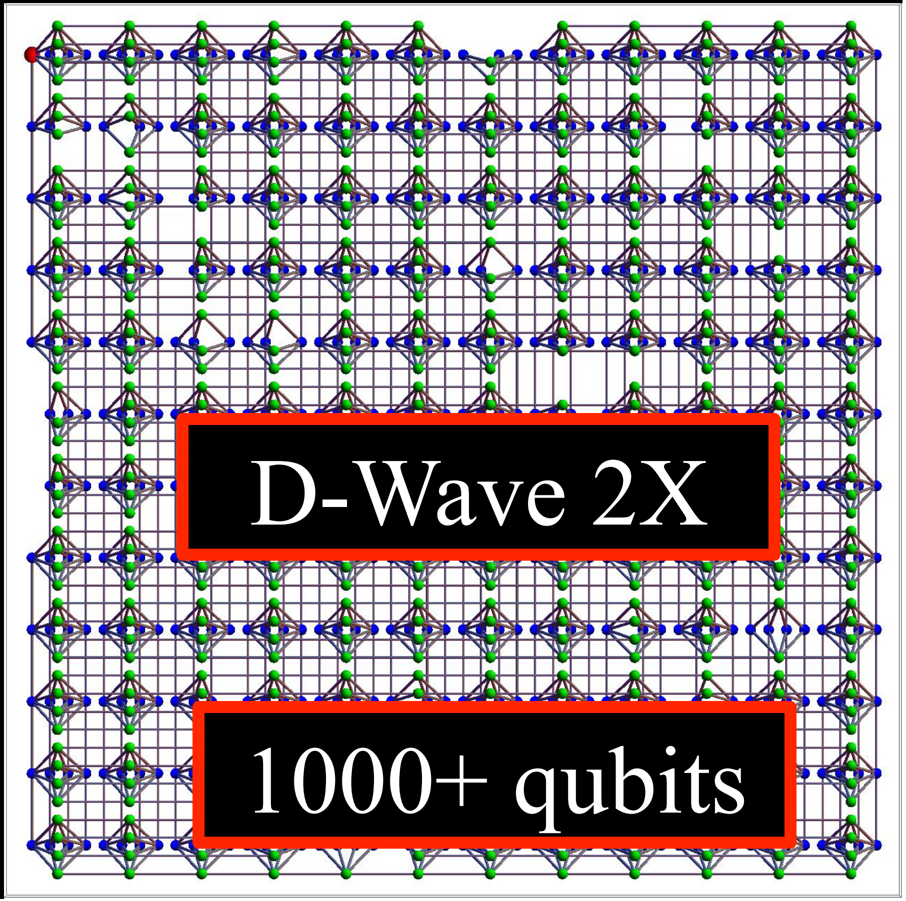
In 35+ years, ***two*** disruptive computing innovations:

Quantum Computing

"Perhaps the quantum computer will change our everyday lives in this century in the same radical way as the classical computer did in the last century."

Nobel press release (Oct. 2012)

David J. Wineland
Serge Haroche



Classical Ising Model

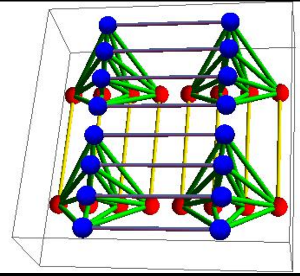
spin = 1/2

Each spin has two values: $s_j = \pm 1$

bias (magnetic) field h_j on each spin

coupling $J_{i,j}$ between spin pairs

$$\mathcal{H}_0 = -\sum_{\langle i,j \rangle} J_{i,j}\sigma_i^z\sigma_j^z - \sum_{i=1}^{N} h_i^z\sigma_i^z$$

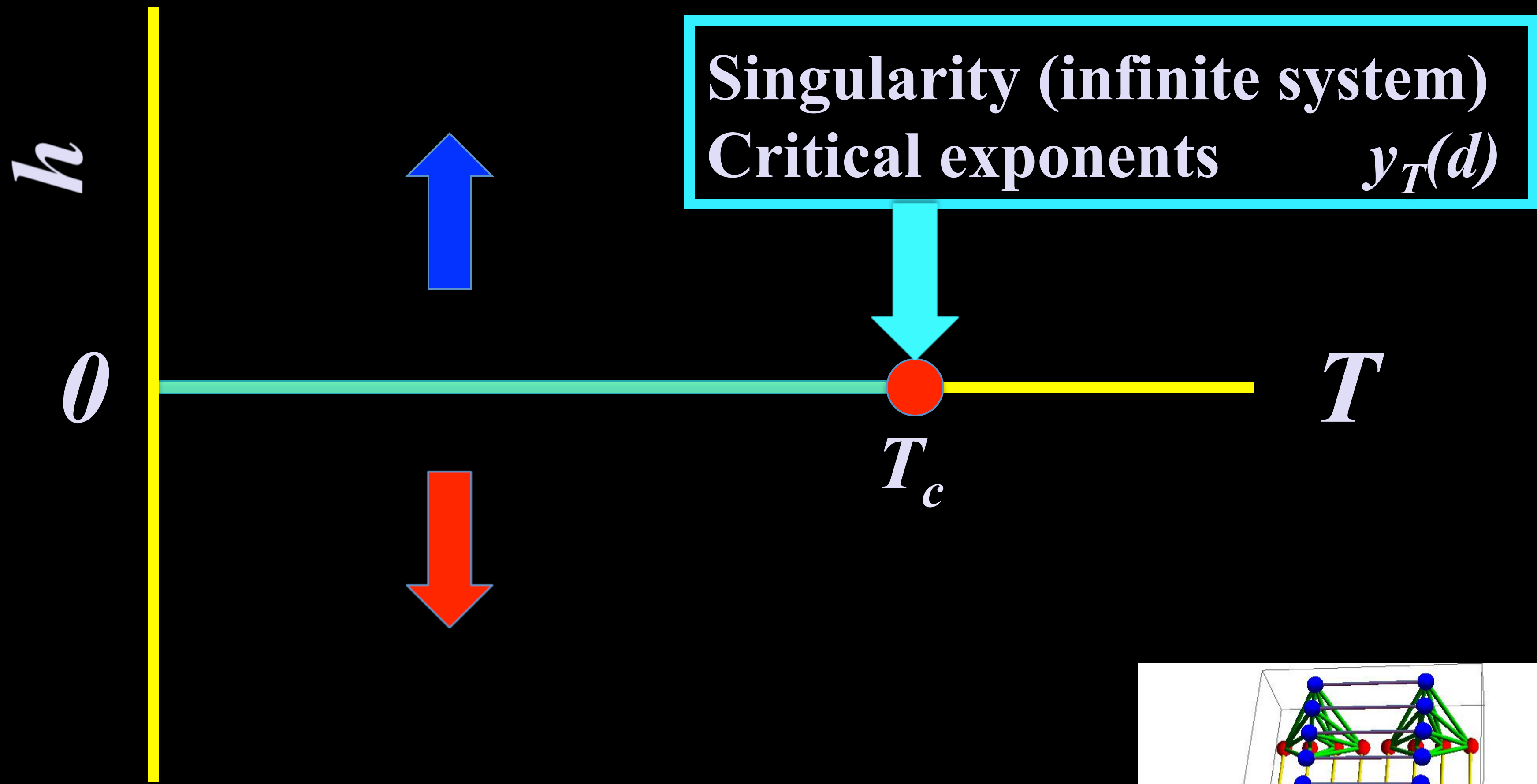


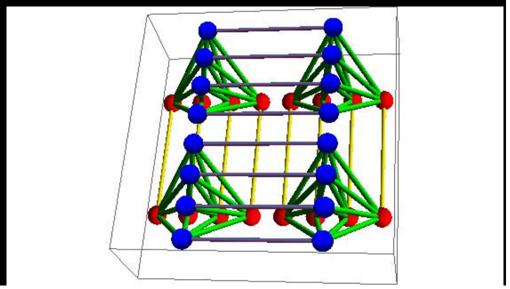
Ferromagnetic:	**all J_{ij} =+1**
Antiferromagnetic:	**all J_{ij} =-1**
Ising spin glass:	**random J_{ij}**
Random field:	**random h_j**

Classical Ising Model

spin = 1/2

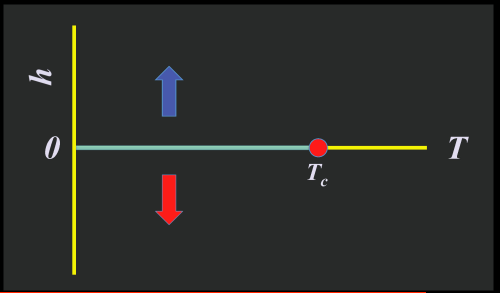
Ferromagnetic: all J_{ij} =+1

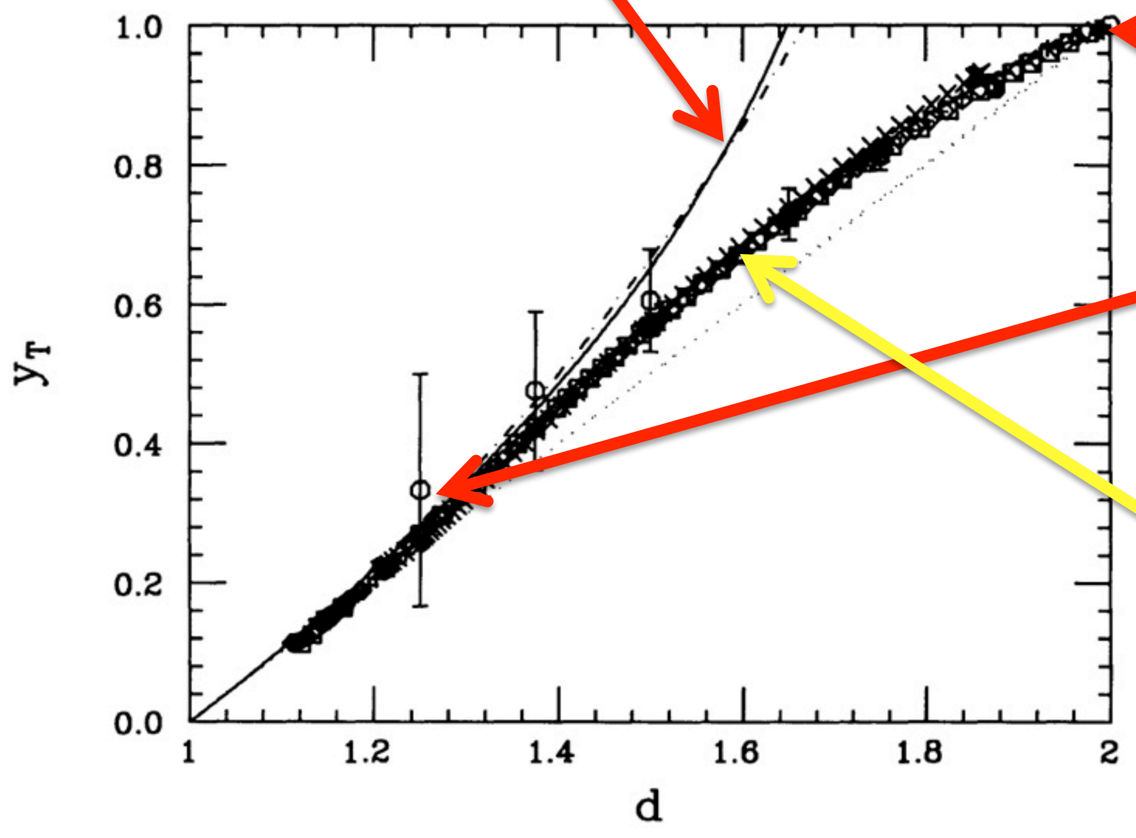




Classical Ising Model

$y_T=1/\nu$ *depends on dimension d*





d=1+ε expansions

ν=1 exact *d=2*

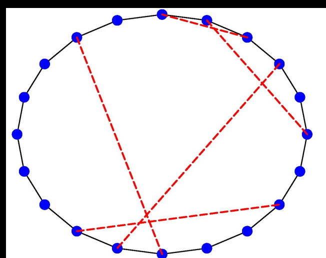
resummed *d=4-ε* expansions

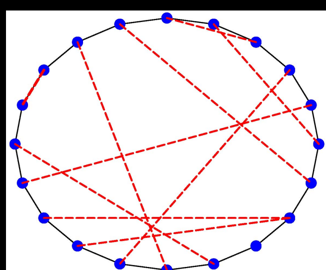
Numerical transfer matrix

Novotny *PRB 1992*

Intro. to Small World Networks

Construct Small-World Networks

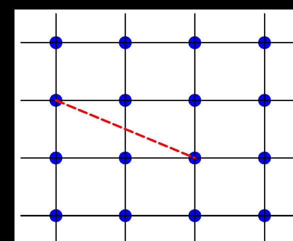


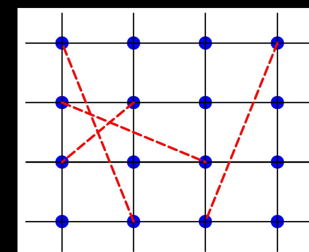


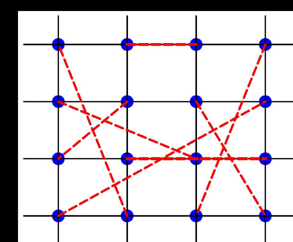
d=1 **to** ***SW***

or

d=2 **to** ***SW***



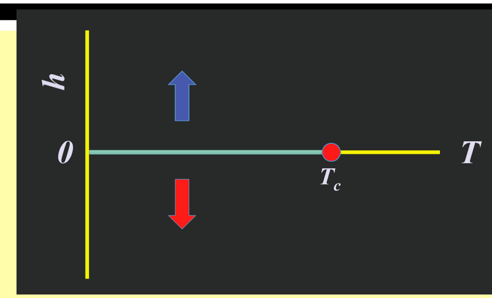




- ***p*** = Fraction of bonds added
- ***L*** (or L^2) *lattice nodes*
- Average distance between nodes ***d~ln(L)***

Small World (*z*-model)

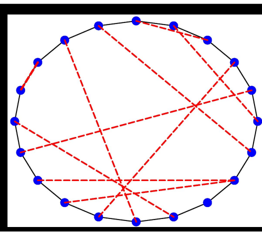
- Start with *d=1* chain of spins
- Randomly connect pairs of spins
- All spins have *z* interactions (strength 1)

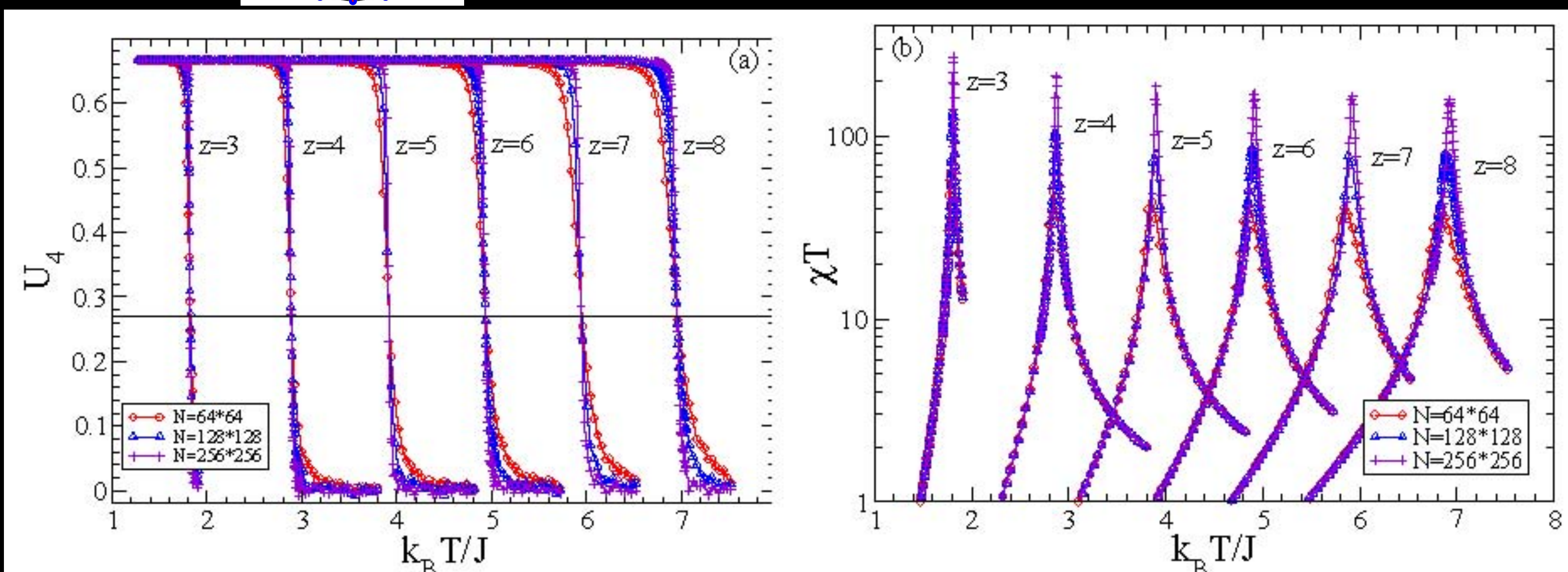


Ising Ferromagnet

Introduced by Scalettar 1991

$z=3$

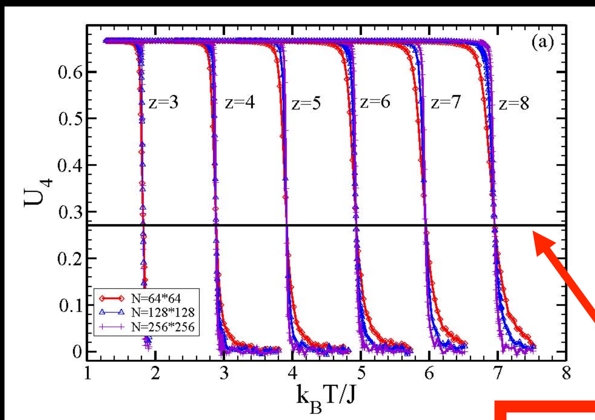




Small-world (*z*-model) y_T mean field

- Binder 4^{th}-order cumulant

Ising Ferromagnet



$$t = |(T - T_c)/T_c|$$

$$U_4(t) = f_U\left(tN^{\frac{1}{2}}\right)$$

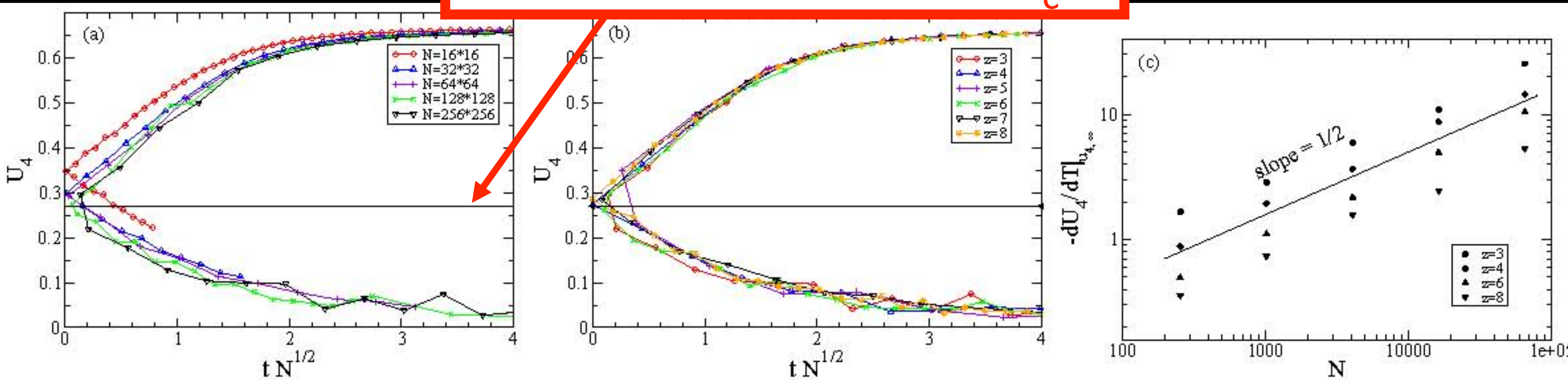
$$\frac{\partial U_4(t)}{\partial t} = N^{\frac{1}{2}}\, f'_U\left(tN^{\frac{1}{2}}\right)$$

Brézin

Zinn-Justin

1985

Exact N=∞ value at T_c



Transverse Field Ising Model

spin = 1/2

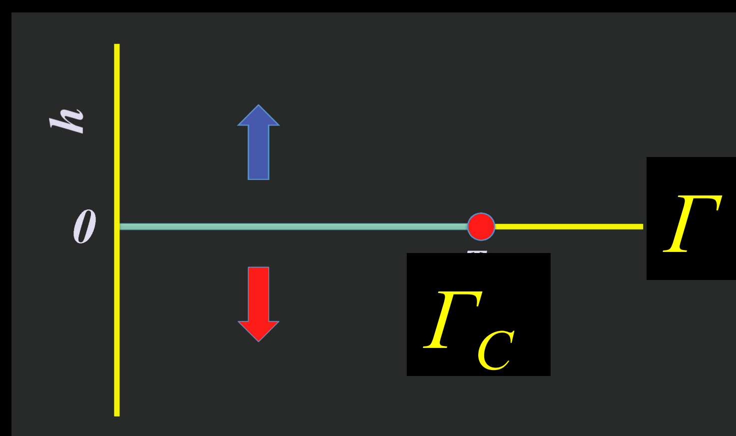
Each spin is a two-component vector: s_j

bias (magnetic) field $h_j\ \sigma_j^z$ each spin

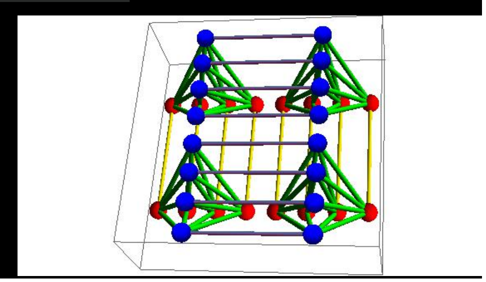
transverse field $\Gamma_j\ \sigma_j^x$ each spin

coupling $J_{ij}\ \sigma_i^z\ \sigma_j^z$ between spin pairs

$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$		$i = \sqrt{-1}$
$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\vec{\psi}_{x+} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	$\vec{\psi}_{x-} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$
$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$	$\vec{\psi}_{y+} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$	$\vec{\psi}_{y-} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$
$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\vec{\psi}_{z+} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\vec{\psi}_{z-} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$



T=0



What can an IDEAL aQC do?

aQC = adibatic Quantum Computer

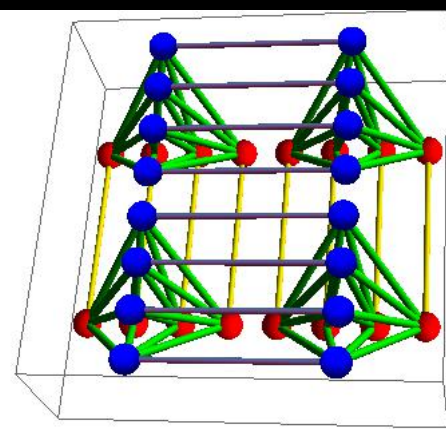
You *give* aQC:

bias h_j on each qubit

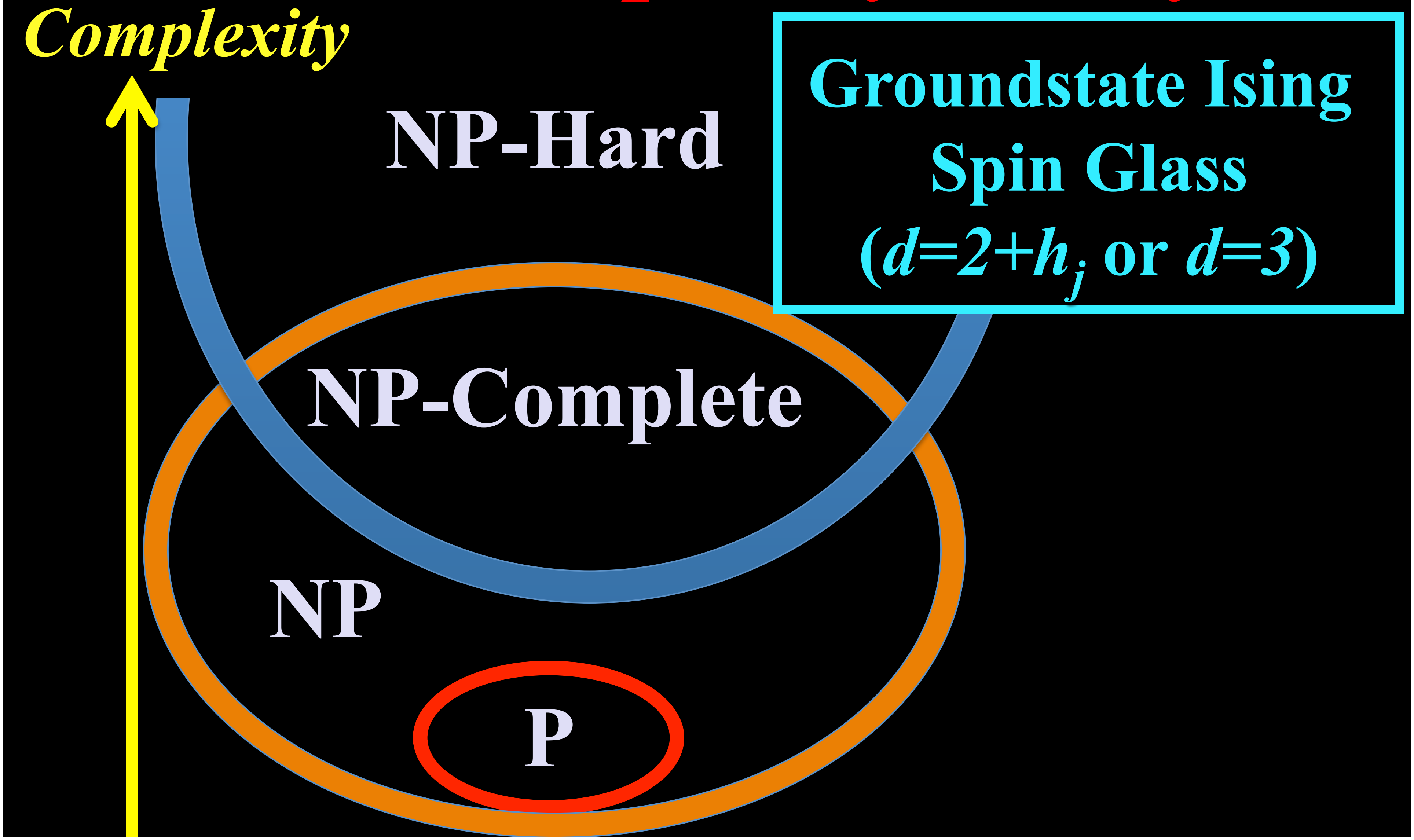
coupling J_{ij} between qubit pairs

You *get* (probably)

Ground state spins $\{s_j\}$



Why should you care?
Complexity theory



SIMD *Parallelization of Ising Spin Glass*

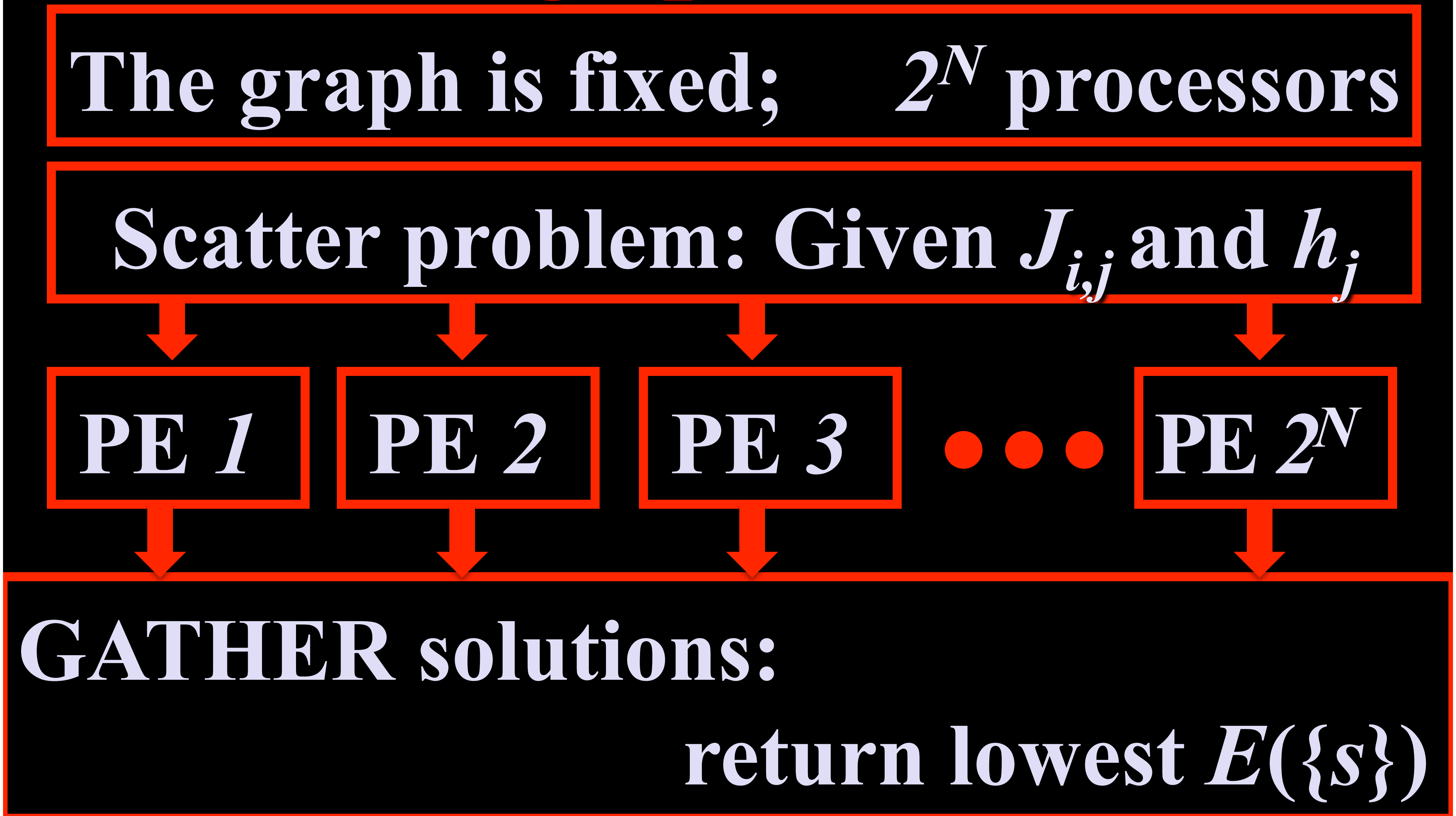
The graph is fixed; 2^N processors

Given $J_{i,j}$ and h_j calculate Energy $E(\{s\})$ for all 2^N configurations

Find Groundstate (lowest energy state)

SIMD = **S**ingle **I**nstruction **M**ultiple **D**ata

SIMD *Parallelization of Ising Spin Glass*



aQC as analog SIMD computer

adiabatic D-Wave Quantum Computer (aDWQC)

Maximum number processors you have parallelized over?

Novotny: Thinking Machine CM-2: 2^{16}=65,536

aDWQC: NASA/Google/USRA: 1000+ qubit: $2^{1000}=10^{301}$

aDWQC 1000 qubit announcement: ***June 22, 2015***

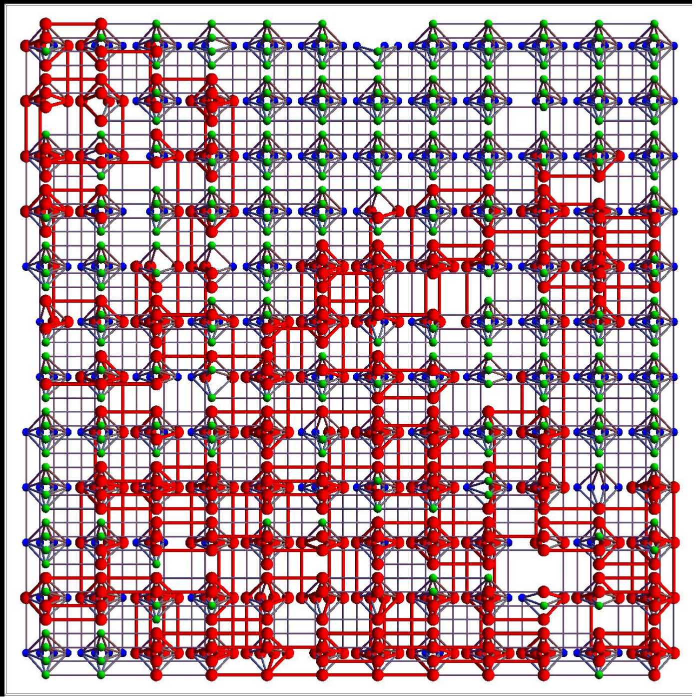
$2^{16} = 7 \times 10^4$

$2^{22} = 4 \times 10^6$

$2^{230} = 10^{69}$

$2^{1000} = 10^{301}$

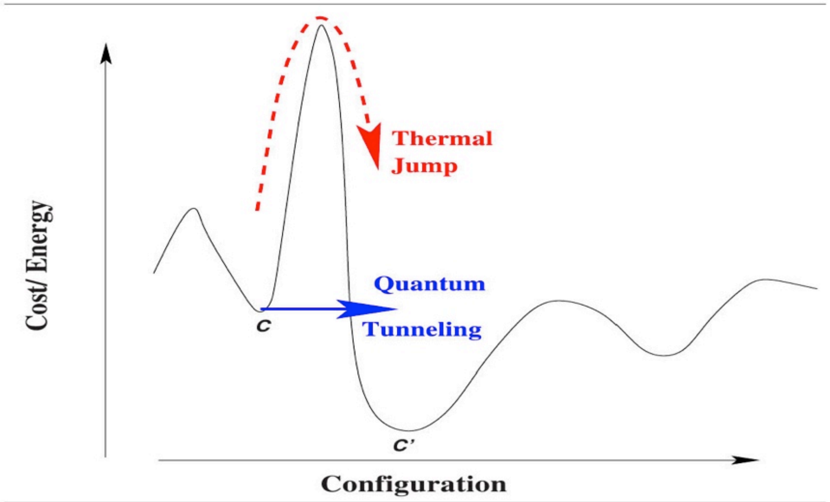
$2^{1097} = 10^{330}$



The Transverse Ising Model

$$H(t)=\Lambda(t)H_0+\Gamma(t)\sum_{i=1}^{N} h_i^x \sigma_i^x$$

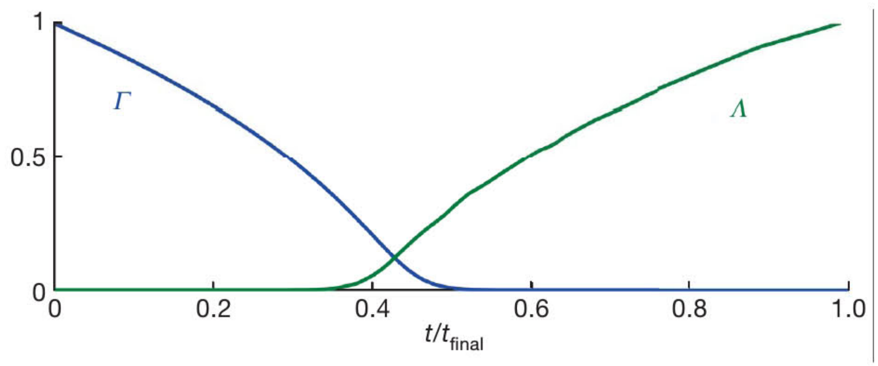
$$H_0=\sum_{i,j=1}^{N} J_{i,j}\sigma_i^z\sigma_j^z+\sum_{i=1}^{N} h_i^z\sigma_i^z$$



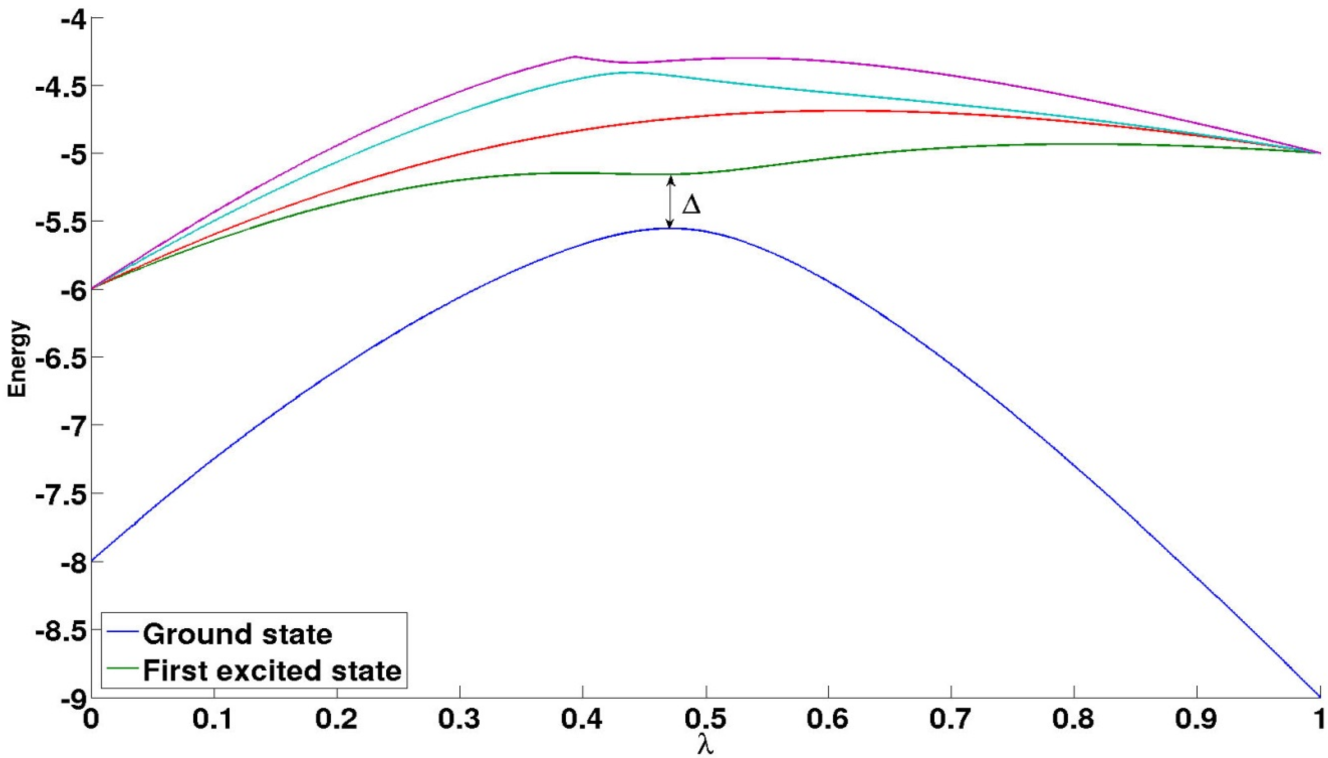
- H_0 is the classical Hamiltonian to be solved
- The Γ(t) allows for quantum tunneling between the classical states
- At t=0 is Γ(0)=1 and Λ(0)=0
- System at t=0 is fully characterized by: $\sum_{i=1}^{N} h_i^x \sigma_i^x$

Annealing

- During annealing the transverse term is slowly turned off ($\Gamma \rightarrow 0$)
- The weight of the Ising Hamiltonian is turned up ($\Lambda \rightarrow 1$)
- If done slowly enough the system should remain in the ground state at all time (adiabatic)



Landau Zener



Laundau Zener Formula: $P_{\text{diabatic}} = \exp\left(-\tau\Delta^2\alpha\right)$ $P_{\text{adiabatic}} = 1 - \exp\left(-\tau\Delta^2\alpha\right)$

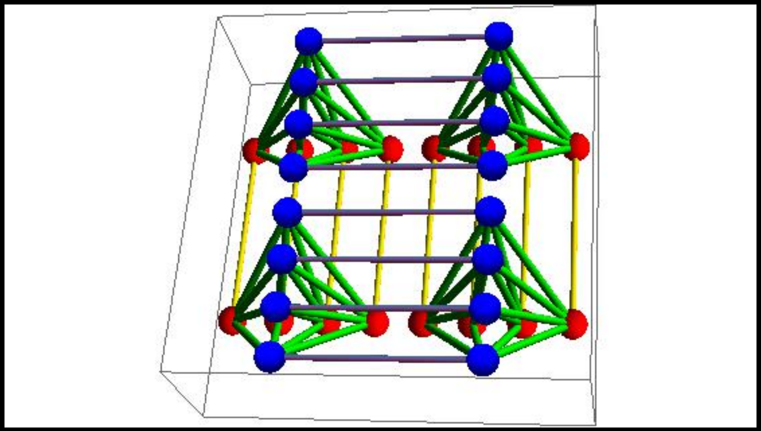
$$\alpha = \frac{2\pi}{\hbar\Gamma(\lambda)}$$

What can aQC do:

aQC = adiabatic Quantum Computer

Gives *global* minimum (ground state) of Ising glass

- (QUBO)
- Limits $T \rightarrow 0$ and $t \rightarrow \infty$
- Probabilistic machine
- On architecture graph



You *give* aQC:

bias h_j on each qubit

coupling J_{ij} between qubit pairs

You *get* (probably)

Ground state spins

Can only quantum physicists program an aQC?

$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$		$i = \sqrt{-1}$
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You *give* aQC:

- **bias h_j on each qubit**
- **coupling J_{ij} between qubit pairs**

You *get* (probably)

- **Ground state spins**

D-Wave 2X 1000+ qubits

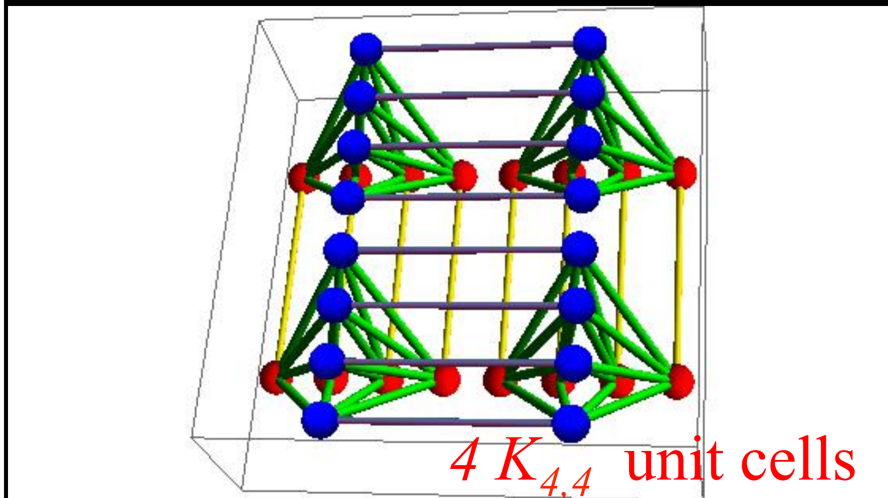
D:Wave

The Quantum Computing Company™

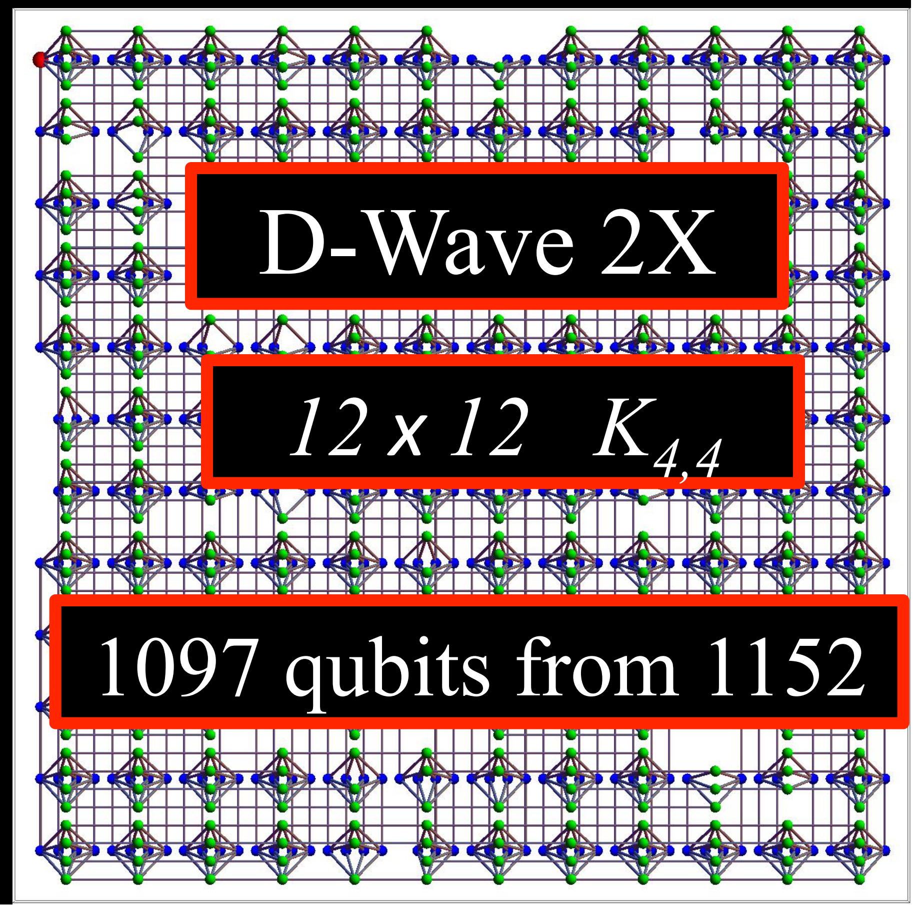
Novotny at D-Wave 2013

D-Wave 2X *1000+ qubits*

$K_{4,4}$ Chimera lattice



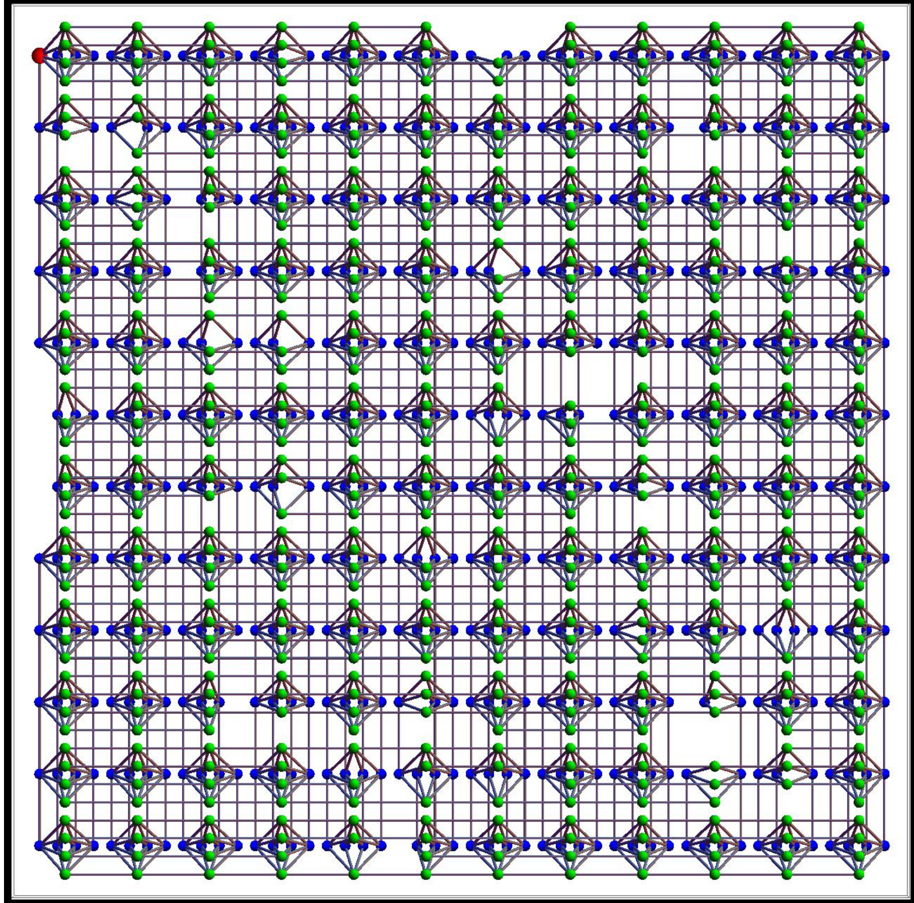
$144\ K_{4,4}$ unit cells

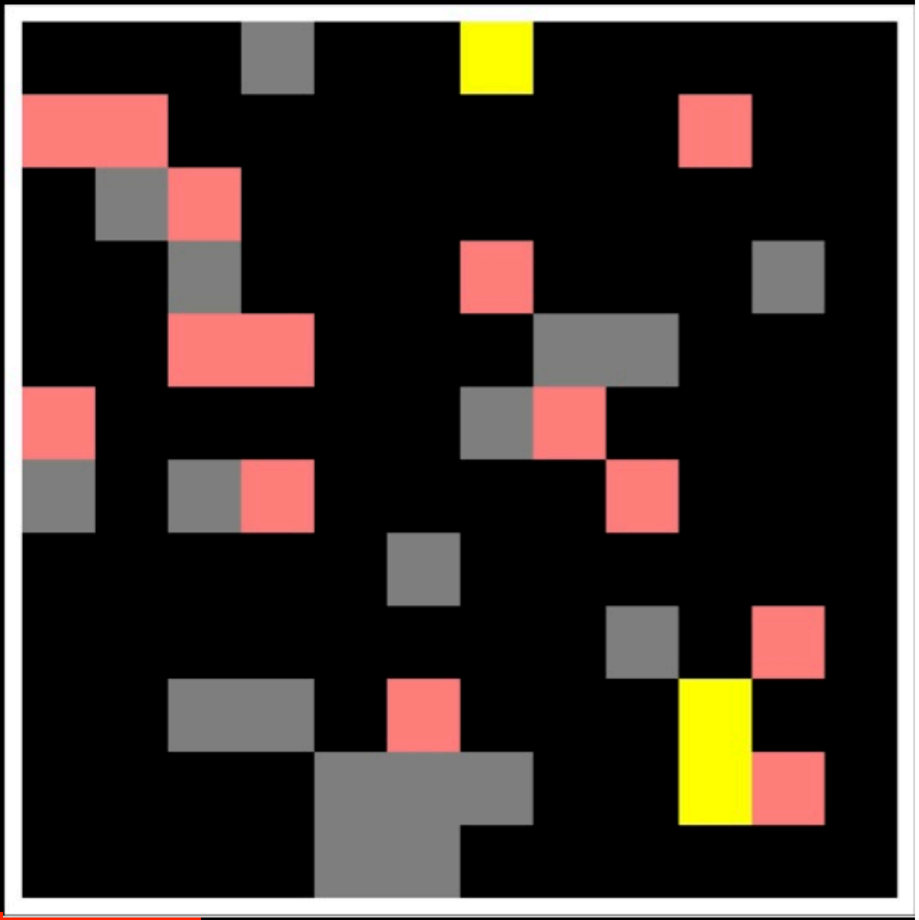


D-Wave 2X *1000+ qubits*

144 $K_{4,4}$ unit cells

$K_{4,4}$ Chimera lattice





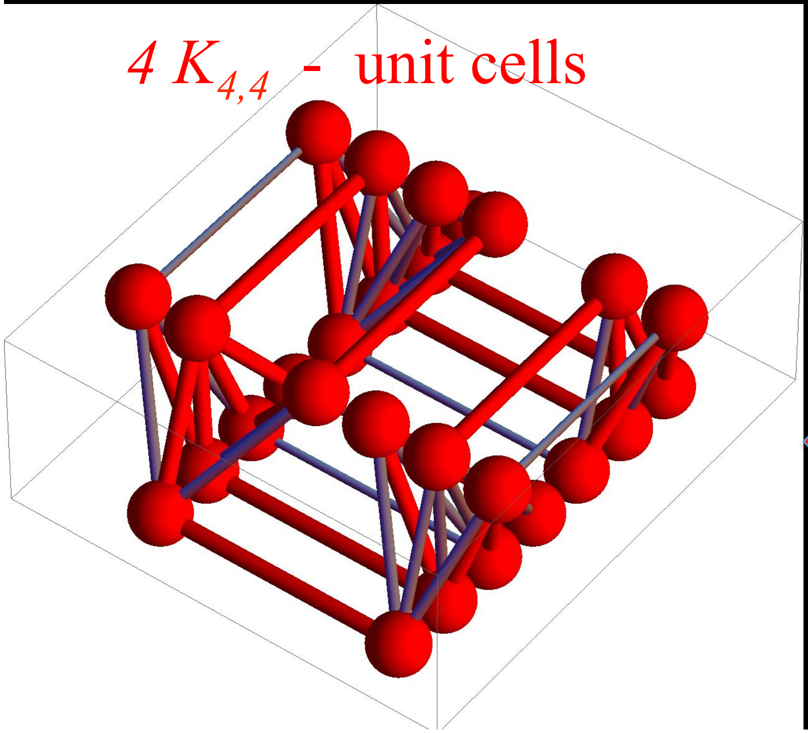
D-Wave 2X

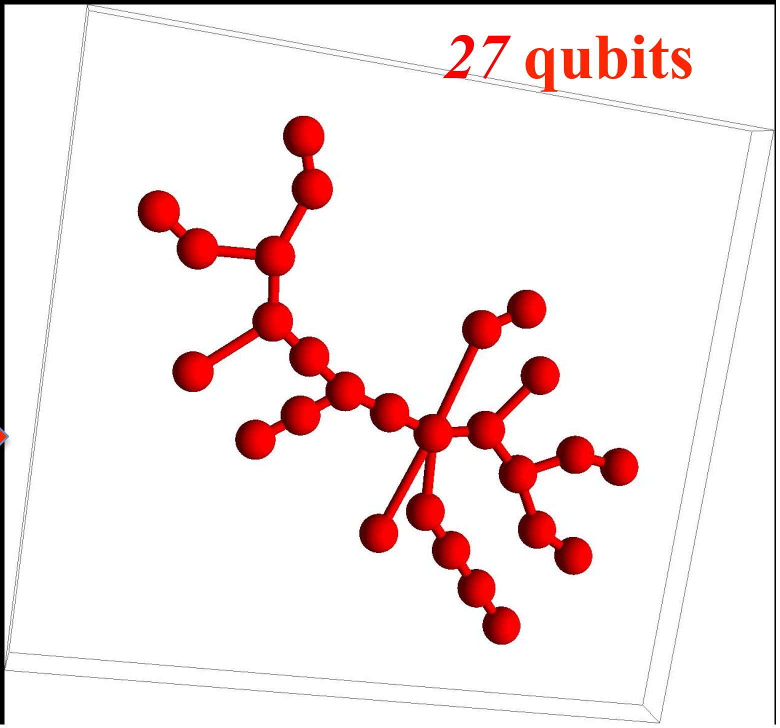
12 x 12 $K_{4,4}$

1097 qubits from 1152; 3060 couplers from 3360

Spanning Trees

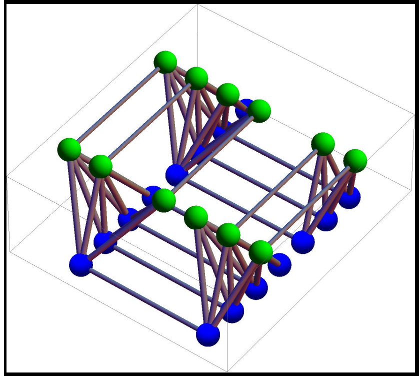
A tree (no loops) that includes every node in a graph

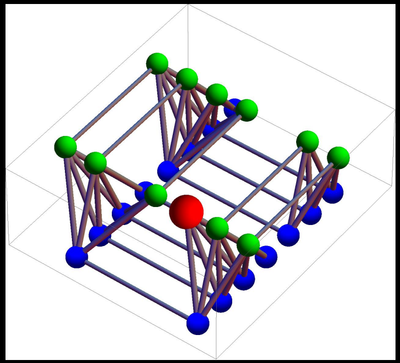


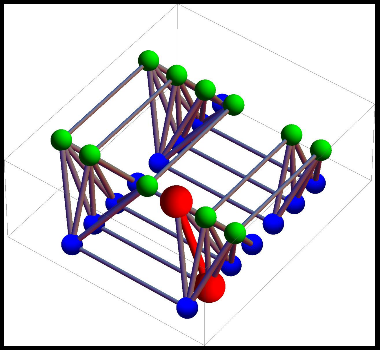


Spanning Trees: algorithm

A spanning tree uniformly drawn from the ensemble of all spanning trees



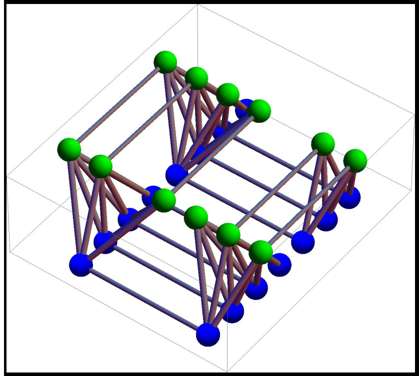
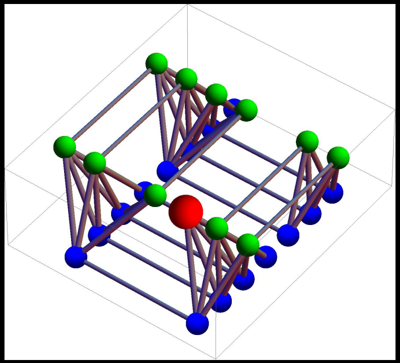
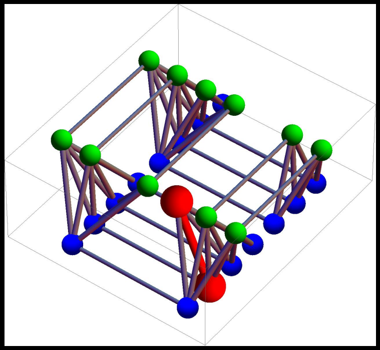
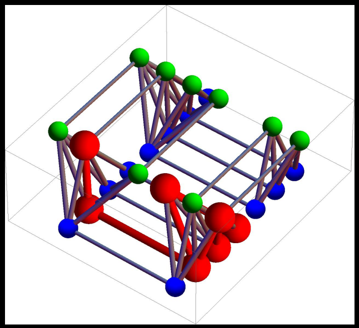
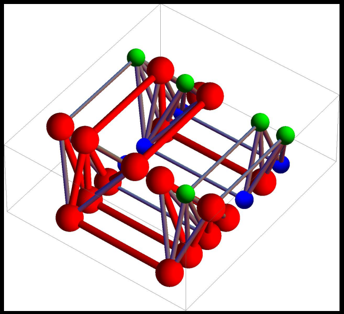




- Randomly choose a node *i*
- Randomly choose a link to another node *j*
  - If node already visited no change to tree
  - If first visit to node *j*, add link & node to tree
- Interchange *j* ➔ *i*

Spanning Trees: algorithm

A spanning tree uniformly drawn from the ensemble of all spanning trees

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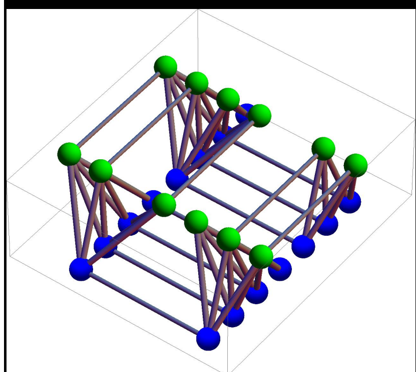
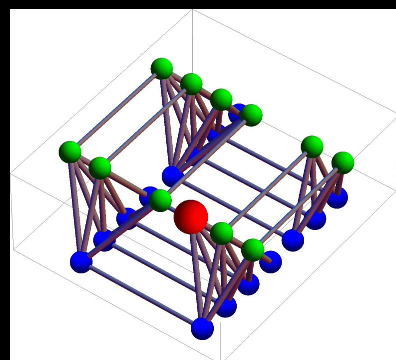
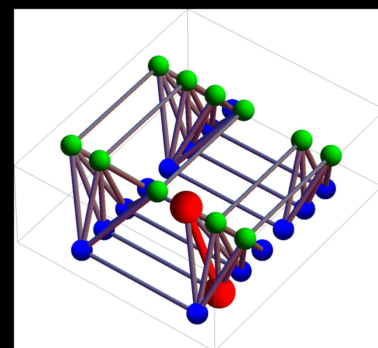
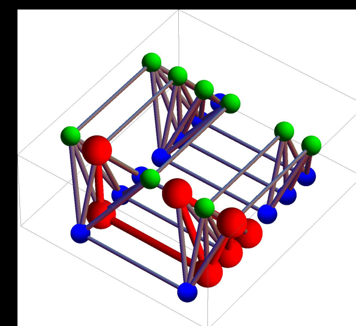
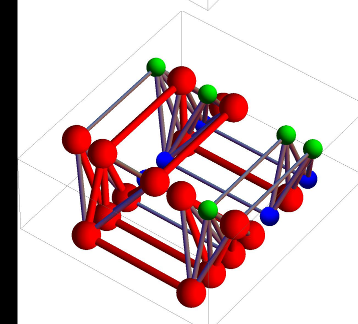
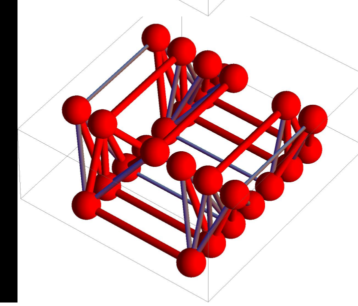
Spanning Trees: algorithm

A spanning tree uniformly drawn from the ensemble of all spanning trees

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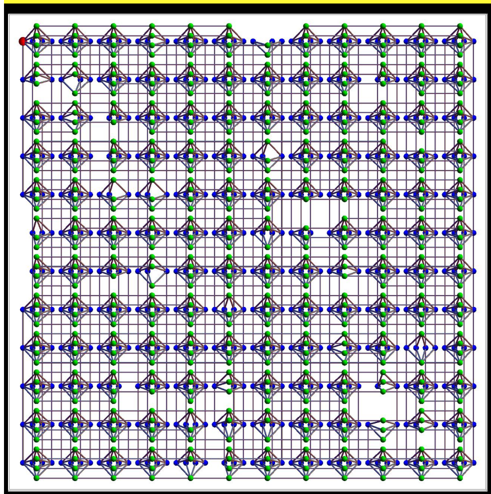
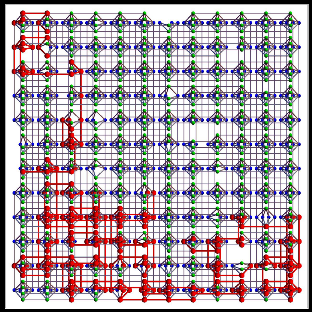
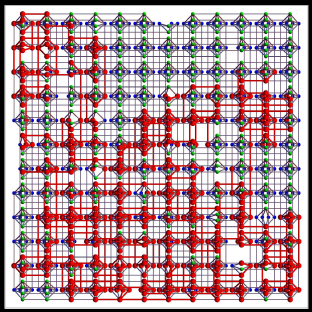
- Randomly choose a link to another node *j*
  - If node already visited no change to tree
  - If first visit to node *j*, add link & node to tree
- Interchange *j* ➔ *i*
- **STOP** when all nodes are in tree

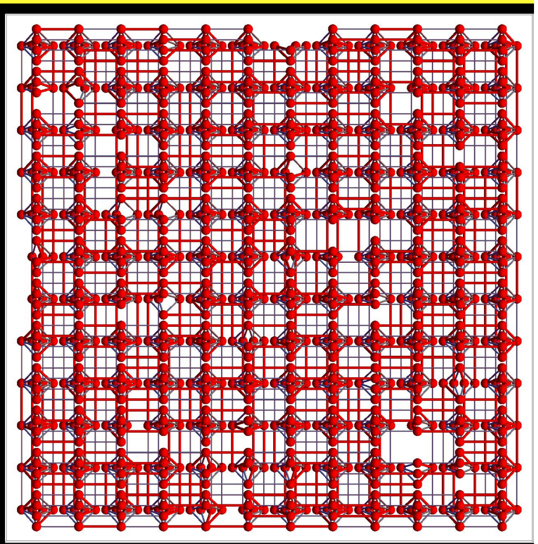
Spanning Trees: algorithm

A spanning tree uniformly drawn from the ensemble of all spanning trees





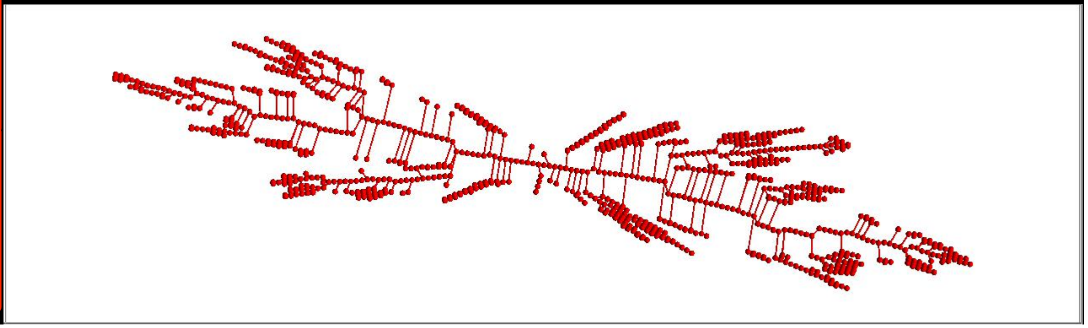




Spanning tree on 1097 qubit
NASA/Google/USRA
D-Wave 2X

$J_{i,j} = \pm 1 \qquad h_j = 0$

$E_{\text{ground}} = N_{\text{qubits}} - 1$

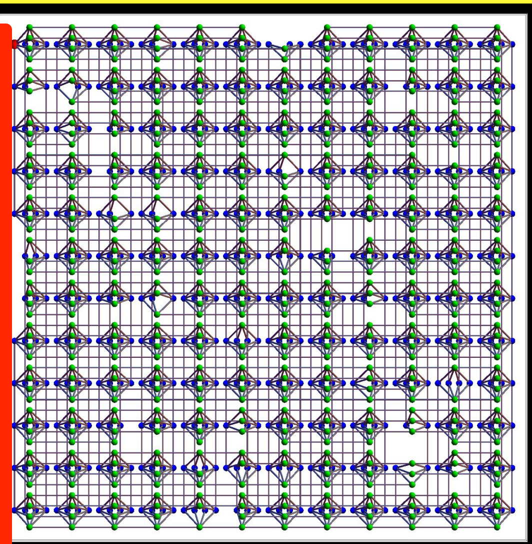


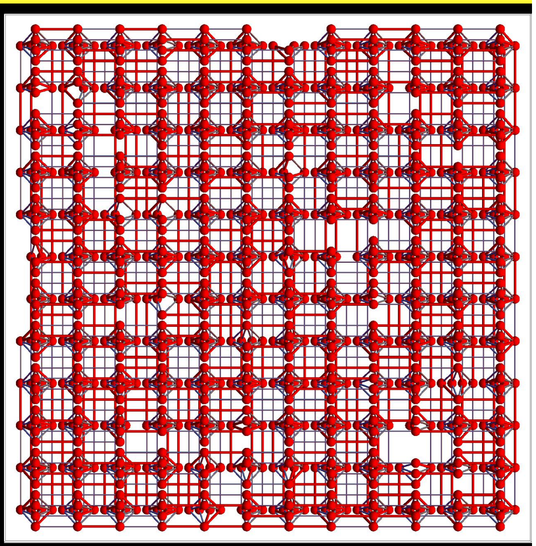
Spanning Trees: algorithm

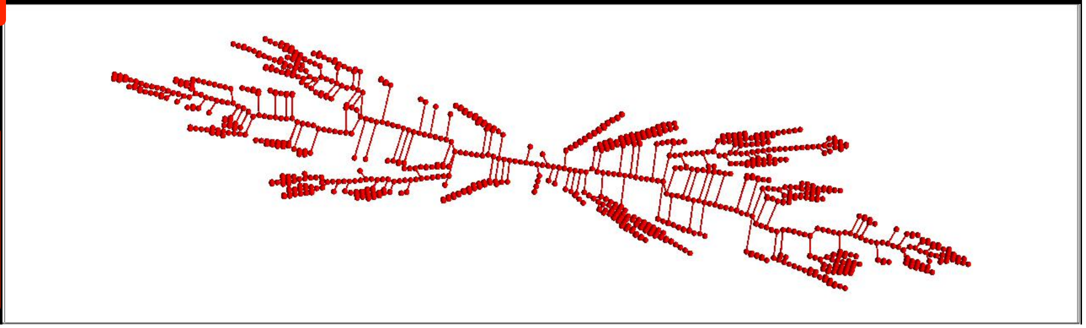
A spanning tree uniformly drawn from the ensemble of all spanning trees

Spanning tree advantage:

- Each tree includes all qubits
- Ensemble covers all bonds
- Ensemble is well defined
- Works on any graph
- Known groundstate
- Known spin arrangements
- Hard problem for aQC







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Recent MSU/JSC aDWQC publication

Available online at www.sciencedirect.com

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28th Annual CSP Workshop on "Recent Developments in Computer Simulation Studies in Condensed Matter Physics", CSP 2015

A Study of Spanning Trees on a D-Wave Quantum Computer

J.S. Hall[a], M.A. Novotny[a,*], T. Neuhaus[b], Kristel Michielsen[b]

[a]*Mississippi State University, 75 B.S. Hood Dr., Mississippi State, MS 39762, USA*
[b]*Institute for Advanced Simulation, Jülich Supercomputing Centre, Research Centre Jülich, D-52425 Jülich, Germany*

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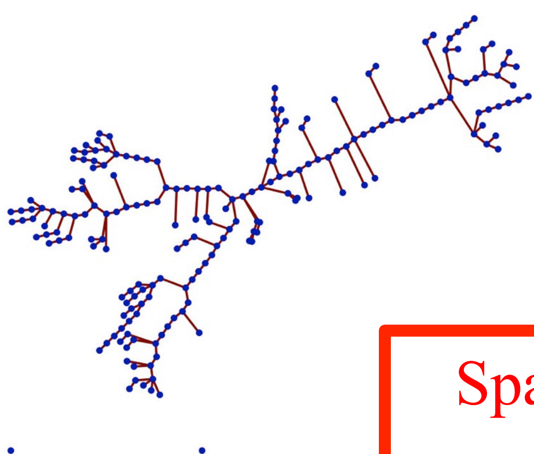
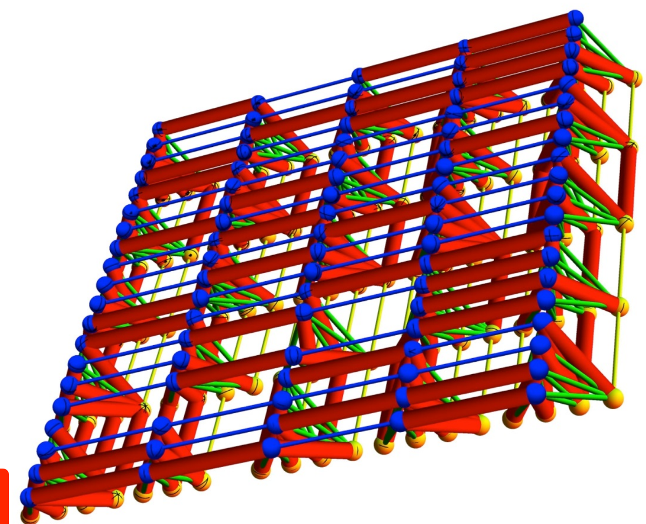




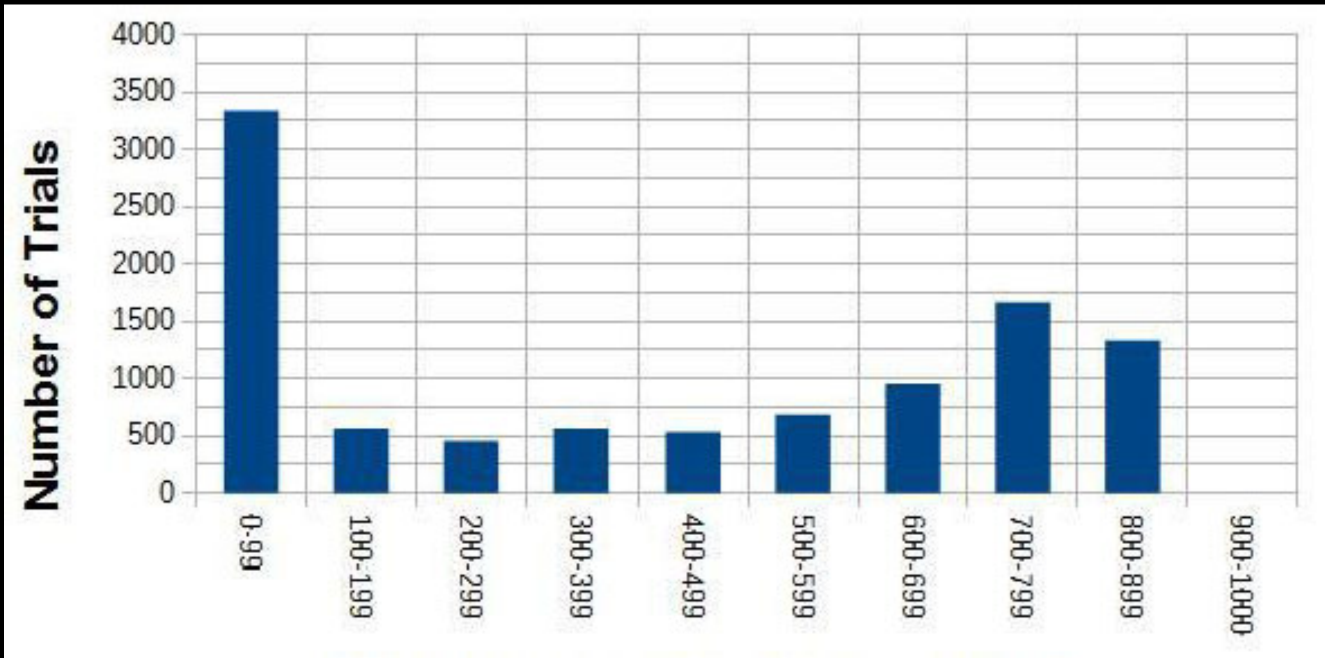
Spanning tree on 496 qubit D-Wave 2

Recent MSU/JSC aDWQC publication

Spanning trees
496 qubit
D-Wave 2

5 x 5 square
197 qubits

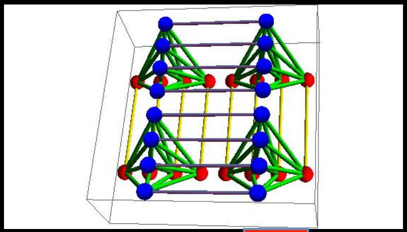
100 spanning trees
100 submissions
each 10^3 anneals
$J_{ij} = \pm 1$



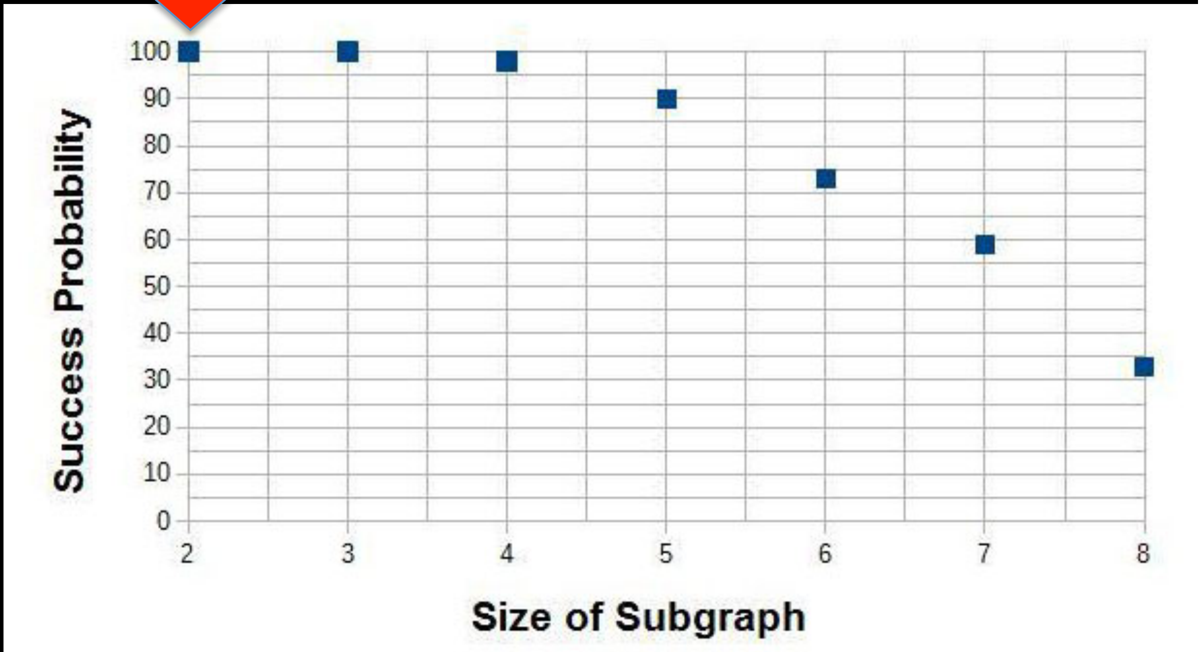
Recent MSU/JSC aDWQC publication

Spanning trees
496 qubit
D-Wave 2

m x m square



100 spanning trees
up to100 submissions
each 10^3 anneals
$J_{ij} = \pm 1$

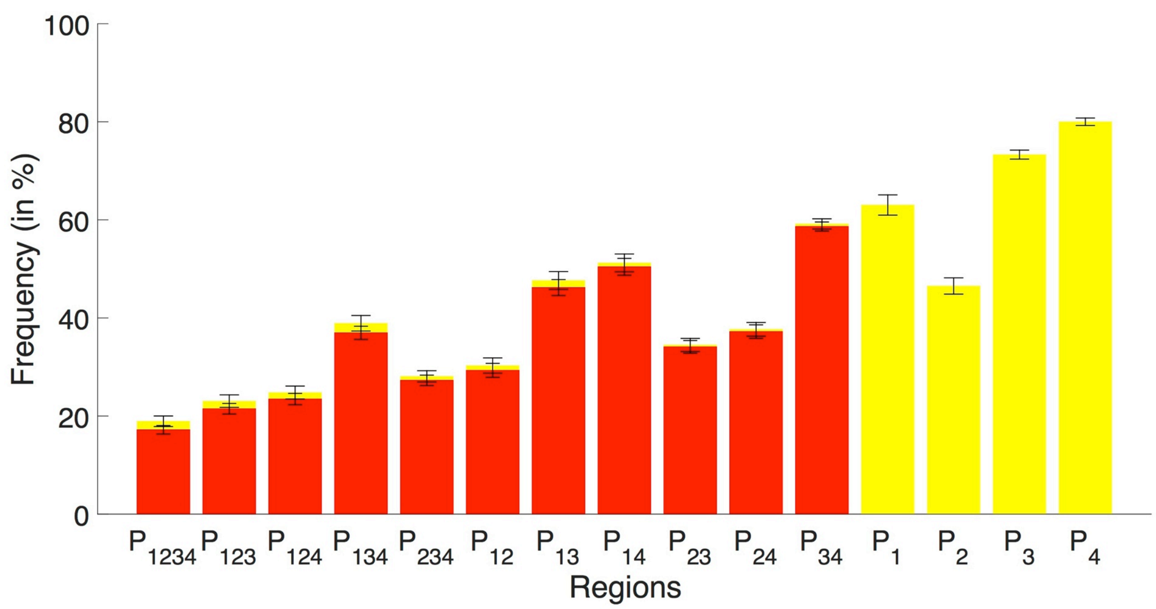


Lukas Hobl D-Wave 2

Spanning trees
476 qubit
D-Wave 2

Are uncoupled subgraphs independent?

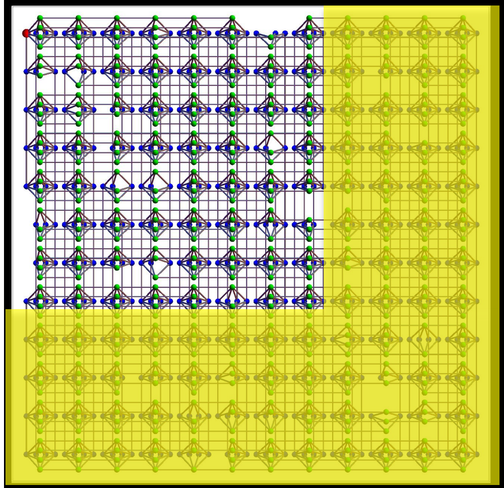
Yes.



1 spanning tree
$J_{ij} = +1$

Day old results: D-Wave 2X

Spanning trees
1097 qubit
D-Wave 2X



Compare D-Wave 2X
With D-Wave 2

Fraction ground state is ever found:

100 submissions, 10^3 anneals per submission

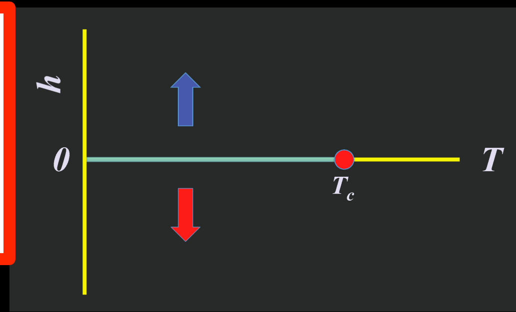
D-Wave 2	33%	100 trees
D-Wave 2X	76%	10 trees

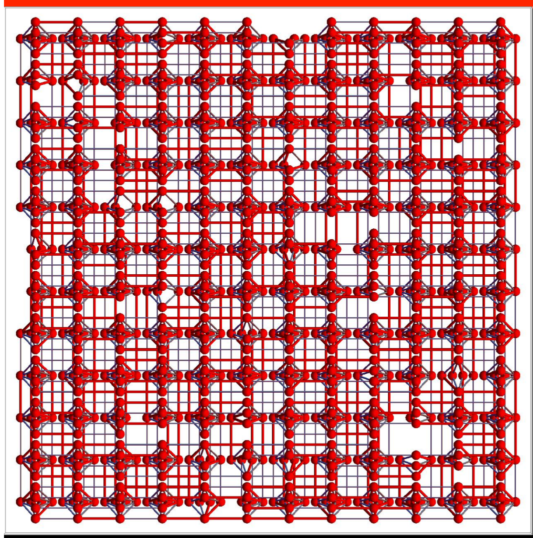
1 spanning tree

$J_{ij} = \pm 1$

Day old results: D-Wave 2X

Ferromagnetic
1097 qubit
D-Wave 2X





Does it find the
ferromagnetic ground
state?

1 spanning tree
$J_{ij} = [-1,+1]$

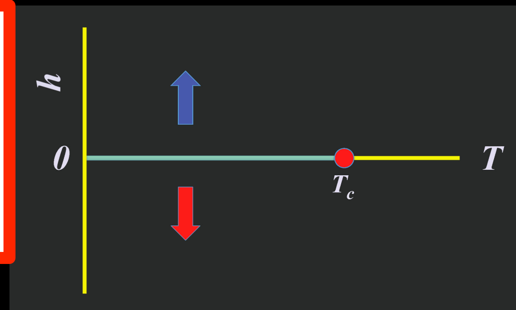
1097 qubit spanning tree:

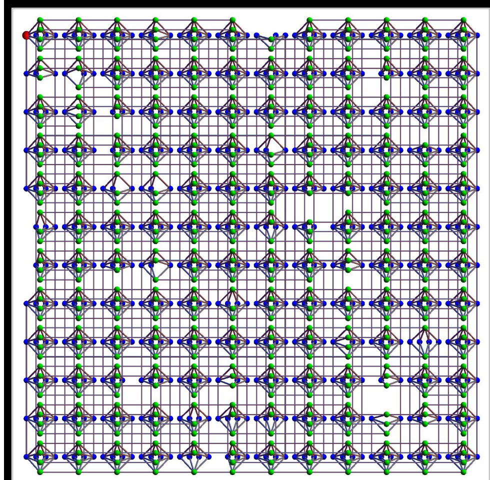
10^3 submissions, 10^3 anneals per submission

Ground state found 4.2% of time

Week old results: D-Wave 2X

Ferromagnetic
1097 qubit
D-Wave 2X





1097 qubit spanning tree:
10^3 anneals per submission; all E=-3060

Does D-Wave find the ferromagnetic ground state?

h=0.0 all s=-1

h=0.06 all s=-1

h=0.07 s=-1, 823 times; s=-1, 177 times

h=0.08 all s=+1

1 full graph
$J_{ij} = +1$

How to improve D-Wave aQC?

Keep anneal time the same
Change function *Γ(t)*

$$P_{\text{adiabatic}} = 1 - \exp\left(-\tau\Delta^2\alpha\right)$$

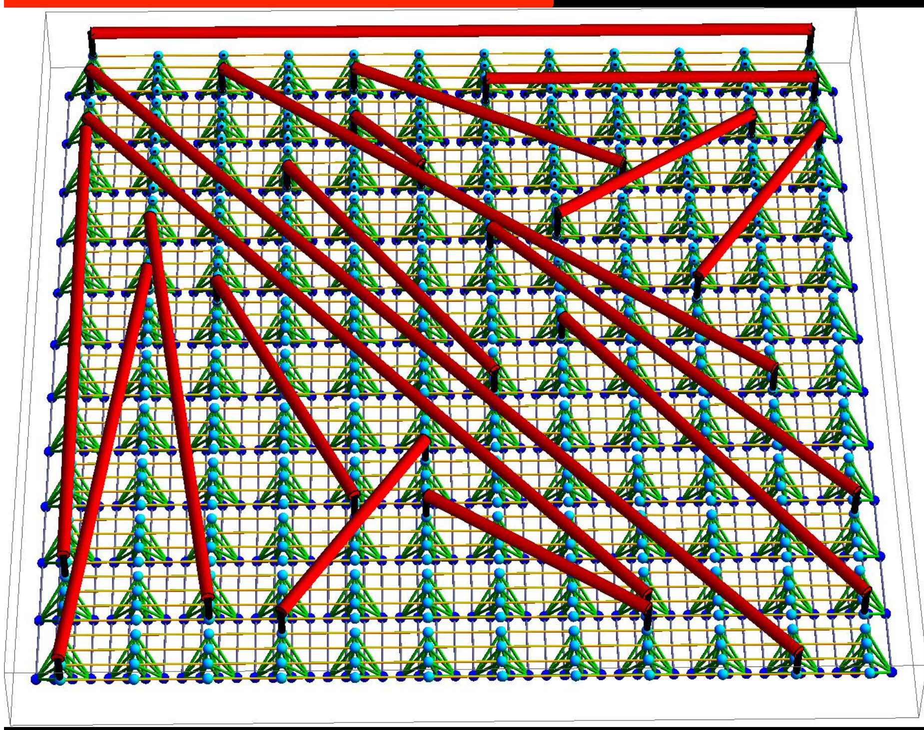
Change success probability

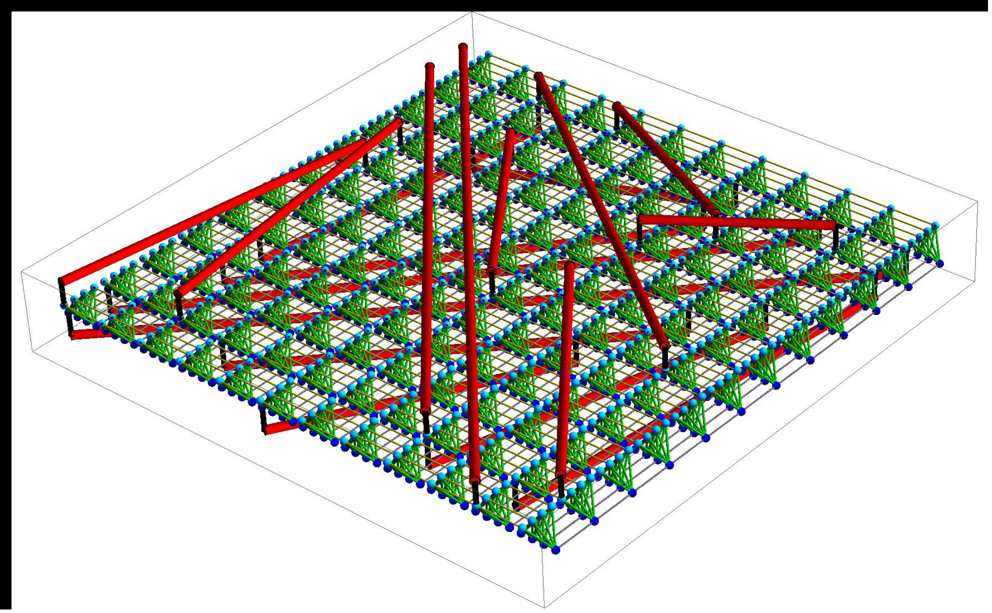
$$P_{\text{adiabatic}} = 1 - \exp\left(-\tau\Delta^{\cancel{2}}\alpha\right)$$

K. Michielsen *et.al.*

How to improve D-Wave aQC?

Change graph
to SW graph
d=∞





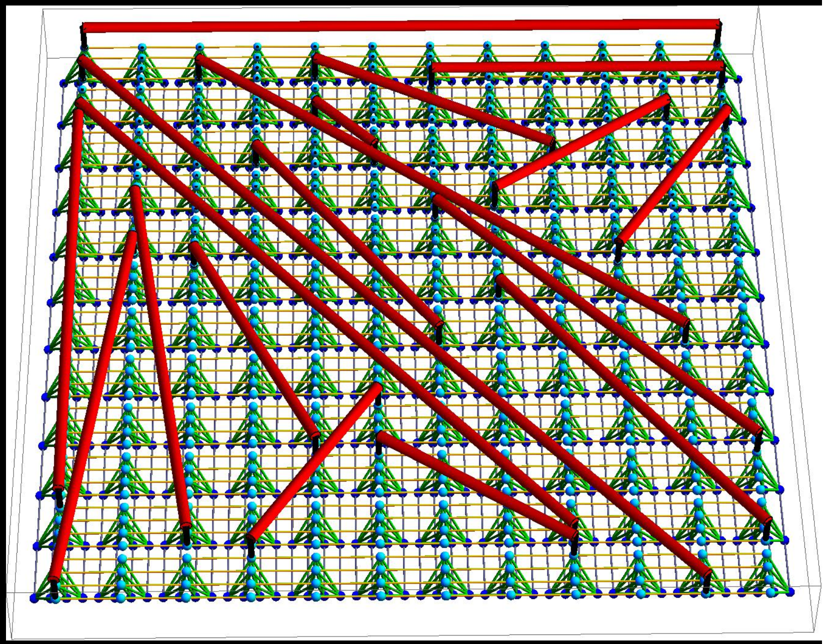
Requires only 1-2
additional chip layers

US Patent Pending

Is aDWQC an aQC?

D-Wave 2X with 1000+ qubits does …

D-Wave 2X is NOT ideal adiabatic quantum computer



More tests needed … more improvements needed

Conclusions and Comments

D-Wave 2X with 1000+ qubits does …

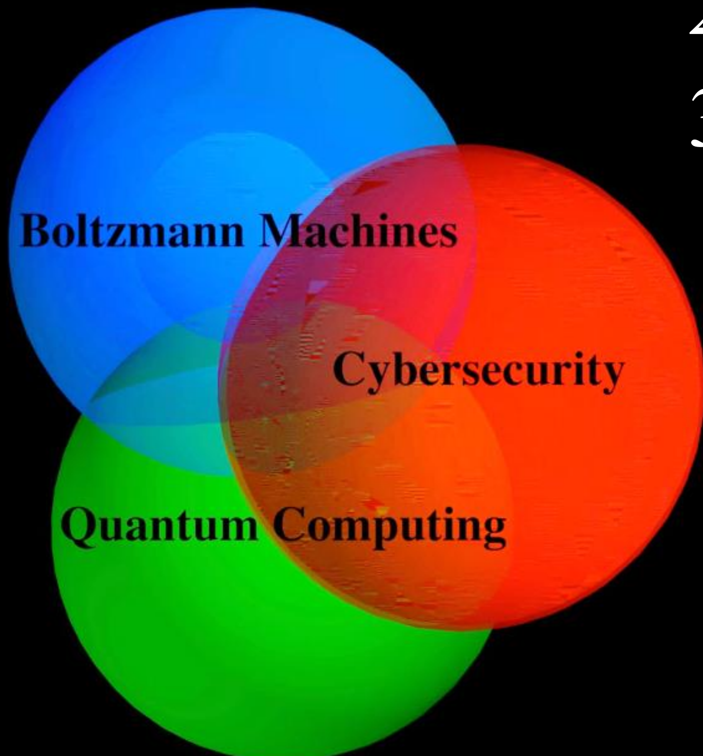
Spanning trees useful as tests of aQC

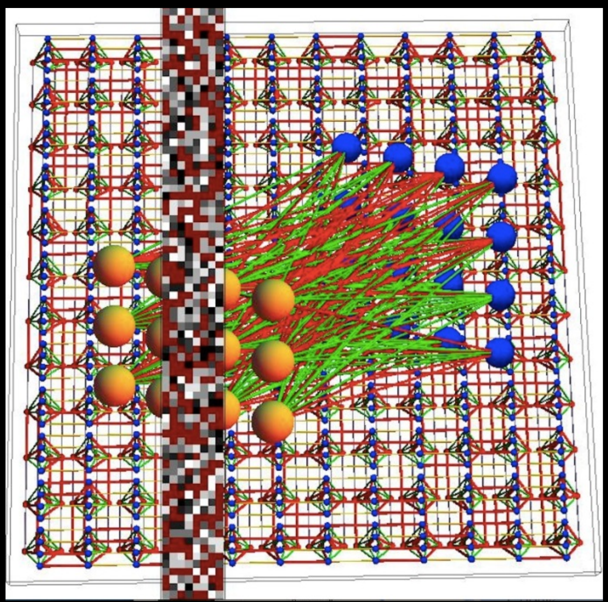
Next generation aQC can be improved

My current use of aQC
Boltzmann Machines (Deep Belief)

Intersection of 3 fields:

1) Cybersecurity
2) Boltzmann machines
3) Quantum computing

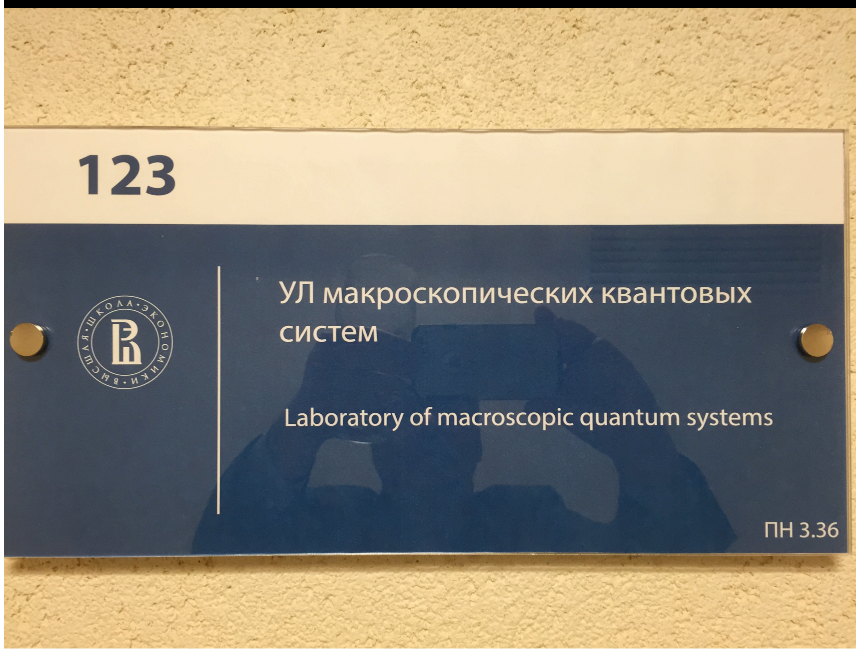




Conclusions and Provocations

Quantum computing is here NOW!

Moscow understands quantum!



Conclusions and Provocations

Quantum computing is here NOW!

Adiabatic quantum computing with 10000+ qubits in four years?

NP-Hard problems are computable!