Numerical analysis in the problem of capacity of systems of densely placed bodies

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Analysis of capacity (conduction) of system of many bodies was started in the works by Maxwell and Rayleigh [1, 2]. Subsequently, significant attention has been paid to the capacity (conduction) of systems of closely spaced bodies, especially after Keller reported [3] that the solutions of Maxwell's and Rayleigh are not valid near the singularity (when bodies "almost touch"). For a long time rigor results in the field were known only for periodic systems of bodies. Recently, rigor analysis was done for the random closely spaced bodies [4, 5]. It has been proven that for closely spaced bodies solution of the boundary value problems for the Laplace equation may be functions which can be found by solving a finite-dimensional problem. This finite-dimensional problem is associated with Delaney-Voronoi network for the system of bodies. This is the core of the so-called network approximation method for continuous problems. The fundamental role in the network approximation method plays strong energy localization effect (shielding Tamm effect [6]). It was found that the network approximation exists not for any system of closely placed bodies, but only for bodies for which the shielding Tamm effect takes place (existence or non-existence of the shielding Tamm effect it determined by the shape of the bodies).

Observation of energy localization effect in systems of closely spaced bodies is difficult, both because of the necessary carry out measurements in very thin gaps between bodies and due to the lack of tools for experimental observation of such physical quantity as energy. The use of computers has allowed to solve this problem and get images of the energy distribution between pairs of bodies and in systems of bodies (for small number of bodies). That is, numerical calculations, which may be done with high accuracy for Laplace equation, were used as a replacement for hard-realizable experiments.

There will be presented new results concerning the fall of the capacity (conductivity) of systems of closely spaced bodies in the presence of defects in the matrix between the neighbor bodies. In this problem, numerical computations are the only possible substitute for experiments. There will be presented comparison of the numerical values and values obtained by the simplified method of [3] for finite (small) values of distance between the bodies and sizes of defects.

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