NUMERICAL MODELING OF COMPLEX GEOPHYSICAL FLOWS IN SHALLOW WATER APPROXIMATION

K. V. Karelsky [1] , A. S. Petrosyan [1,2]

[1] Space Research Institute of the Russian Academy of Sciences

[2] Moscow Institute of Physics and Technology (State University)

Shallow water approximation is used widely to study the large-scale processes in Earth and planetary atmospheres, in oceans. The main difficulty in numerical simulation of nonhomogenous shallow water equations consists in their nonlinearity and their non-divergence property determined by nonhomogeneity of the right-hand side of the momentum conservation equations due to bed complexity. The presence of a non-divergent term induces highly nonlinear effects caused by stepwise change of hydrodynamic quantities in the areas of its sharp change in addition to nonlinear phenomena due to hyperbolic structure of shallow watyer equations. The other problem consists in compatibility of solutions of traditional depth-averaged equations with depth-averaged solutions of initial Euler equations due to significant role of shallow flows dependences on vertical coordinate. Numerical methods have been developed and effectively used in studies of shallow water flows on complex boundary when an external force effects are insignificant.

In present work we propose to use the Riemann-solver which is adapted to the flow parameters for calculating shallow water flows over an arbitrary bed in the presence of external force. The proposed method belongs to the family of methods based on the solution of the dam-break problem. The method consists in reducing of the problem to successive solutions of classical shallow water equations on the flat plane using Godunov method with allowance for the vertical nonhomogeneity effect in calculating the fluxes through the boundaries of cells adjoining to stepwise boundaries. The vertical nonhomogeneity leads to the Riemann problem solution on a step based on the quasi-two-layer shallow water model. Quasi-two-layer model is extended here for a generalized time-dependent bed which represents an external force. The term time-dependent bed means that at each time moment bed can have different values. We are solving the shallow-water equations for one layer, introducing the fictitious lower layer only as an auxiliary structure in setting up the appropriate Riemann problems for the upper layer. Besides quasi-two-layer approach leads to appearance of additional terms in one-layer finite-difference representation of balance equations. These terms provide the mechanical work made by nonhomogeneous bed interacting with flow.

The main difficulty in modeling of fluid flows over a complex bed consists in that both partly and complete flooded domains may take place. Partly flooded domains present a real challenge for most finite-difference schemes and require special efforts need to be made to capture such domains. Algorithm suggested in our work avoids this difficulty in a natural way. We extend algorithms to the case of multiply-connected domains including partly flooded and dry regions. Partly flooded domains may appear in flows when fluid depth is related to the value of bed gradient when approximated by steps. In this case the step wall is partly wetting and the algorithm suggested in present work considers exactly mechanical work done only by this part of the step. Mechanical work done by nonhomogeneous bed means that the work is done in the process of interaction of non-stationary flow with nonhomogeneous bed, with the step and with part of the step. Numerical simulations are performed based on the proposed algorithm of various physical phenomena, such as a breakdown of the rectangular fluid column over an inclined plane, large-scale motion of fluid in the gravity field in the presence of Coriolis force over an mounted obstacle on underlying surface. Computations are made for two dimensional dam-break problem on slope precisely conform to laboratory experiments. Interaction of the Tsunami wave with the shore line including an obstacle has been simulated to demonstrate the effectiveness of the developed algorithm in domains including partly flooded and dry regions.